Paper ID: FRC 830.005 | Version: v0.2

FRC and Quantum Born Reciprocity v0.2: A Present-Structure Obstruction to Majid-Style Self-Duality

Author: Hadi Servat

Affiliation: Independent Researcher

Contact: publish@fractalresonance.com

Website: fractalresonance.com

Abstract

> *FRC has an exact normalized entropy-coherence ledger involution $D(s,y)=(-y,-s)$, a typed category of operational registers, and now both one-parameter and full-qubit lift obstructions. This paper asks whether those results constitute Born reciprocity or Majid-style representation-theoretic self-duality. They do not. The canonical Born exchange $B_a(q,p)=(ap,-q/a)$ is order four and symplectic, whereas D is order two and anti-symplectic on the ambient ledger plane; indeed their different orders prevent conjugacy even by a bijection. The ledger datum also does not reconstruct a state-space action, and FRC 830.004 shows that this non-reconstruction occurs in natural qubit*

> *fibers. A primitive Hopf algebraization makes D only a Hopf automorphism of one chosen ledger algebra, without an independently defined dual object, pairing, semidualisation, representation exchange, or physical state-observable map. The general one-parameter identity-lift obstruction excludes every nonidentity lift under its monotonicity hypotheses, while the full qubit ledger excludes incoherent operations, unital qubit CPTP channels, and unitary or antiunitary Wigner symmetries as global lifts. A general nonunital, coherence-generating CPTP lift remains open. Present FRC therefore has exact coordinate, typed-morphism, dimension-threshold, and scoped no-go mathematics, but not a Born- or Majid-style self-duality.*

0. Result in one paragraph

Present FRC reciprocity is not quantum Born reciprocity or a Majid-style self-duality. The conclusion is structural, not rhetorical. The canonical Born exchange and the FRC ledger involution have different composition and orientation behavior. More decisively, the ledger datum (M_+, ω, D) does not determine an action on the states that generate its coordinates: inequivalent state involutions with different fixed sets can induce the same D. One may place a standard primitive Hopf algebra on the ambient ledger coordinate ring, where pullback by D is a Hopf automorphism, but that construction is inherited from the additive coordinate plane and supplies neither a dual physical object nor a Hopf pairing, semidualisation, representation exchange, or observable-state operation. In every one-parameter arena satisfying the monotonicity hypotheses of FRC 830.004, D has no nonidentity lift. The full qubit ledger is the first arena where the question is dimensionally nontrivial, but incoherent operations, unital channels, and Wigner symmetries each fail as global lifts. FRC therefore retains an exact P2 ledger symmetry, typed P3 lift criteria, and scoped lift obstructions; promotion to Born or Majid self-duality is rejected for the present evidence base.

1. Question, dependency, scope, and status

The FRC 830 series began by separating coordinate transforms, bookkeeping identities, balance laws, dualities, and self-dualities. It then established:

1. a complete affine classification of reciprocity-preserving ledger maps;
2. a unique nonidentity exact form-reversing involution D on the

nonnegative-entropy cone within that class;

1. a typed category in which a strong morphism must act on states and commute

with the ledger map; and

1. an exact obstruction preventing D from lifting nontrivially to the

fixed-mean von Mises family on an open domain;

1. a general one-parameter identity-lift obstruction in normalized ledger

coordinates; and

1. an exact qubit-ledger image, a natural fiber non-reconstruction theorem,

and three operation-class no-go results.

The present paper asks the comparison question that those results now make answerable:

> *Do the structures actually established through FRC 830.004 satisfy the mathematical requirements of Born reciprocity or Majid-style representation-theoretic self-duality?*

The answer is no. This is a **present-structure obstruction**, not a universal impossibility theorem. FRC 830.004 has now located the exact dimension threshold and proved several physical-lift obstructions. It leaves a general nonunital, coherence-generating CPTP construction genuinely open. A future independently defined algebraic or open-system structure could therefore reopen the comparison, but the current ledger cannot be counted as evidence for that future structure.

The proof status is exact-mathematical candidate pending independent review. The paper introduces no new physical observable, quantum group, experiment, or empirical validation. It does not alter the operational status of the canonical FRC relation

$$dS + k^* d \ln C = 0. \tag{1}$$

This manuscript integrates the completed internal result of FRC 830.004. It remains a working draft pending independent proof review and is not yet a journal-facing paper.

1.1 Dependency record

Source	Result used here	Review status

FRC 830.000	Five claim classes and the structural promotion test.	Framework terminology.
FRC 830.001	Affine classification and uniqueness of D on M_+.	Internal algebra review complete; independent review pending.
FRC 830.002	Weak/strong register morphisms and commuting-square lift criterion.	Internal formal review complete; independent review pending.
FRC 830.003	Exact von Mises no-open-domain lift theorem.	Internal analytic and numerical review complete; independent review pending.
FRC 830.004	General one-parameter obstruction, exact qubit ledger image, natural qubit-fiber non-reconstruction, and operation-class no-go theorems.	Internal analytic and deterministic review complete; independent review pending.

2. The structures being compared

The word *reciprocity* occurs in several legitimate but inequivalent mathematical settings. A comparison must begin by naming their objects and maps.

2.1 The current FRC ledger datum

The canonical k^* is the scale-invariant Boltzmann bridge. Its numerical representation k^*_μ is determined by the units of the declared information register μ. For the natural-information representation used in this paper, take $\mu = \mu_{\mathrm{nat}}$ and $k^*_{\mu_{\mathrm{nat}}} = 1$, and define

$$x = \frac{S_{\mu_{\mathrm{nat}}}}{k^*_{\mu_{\mathrm{nat}}}}, \qquad y = \ln C_{\mu_{\mathrm{nat}}}, \qquad M_+ = [0,\infty) \times (-\infty, 0]. \tag{2}$$

A lawful entropy-unit conversion multiplies S_μ and k^*_μ by the same predeclared positive factor and leaves C_μ unchanged. It therefore leaves (x, y) unchanged. The comparison below concerns the normalized ledger datum, not an independently evolving bridge or a dynamics on scale space.

The current exact ledger datum is

$$\mathcal{L} = (M_+, \omega, D), \qquad \omega = dx + dy, \qquad D(x,y) = (-y, -x). \tag{3}$$

Within the affine, invertible, kernel-preserving automorphisms of M_+, FRC 830.001 proves

$$D^2 = I, \qquad D^*\omega = -\omega, \tag{4}$$

and proves that D is the unique nonidentity involution in that stated class. The entries x and y are derived scalar channels. Equation (3) by itself contains no state space, observable algebra, coproduct, duality pairing, representation category, Hamiltonian, symplectic form, or environment.

2.2 Canonical Born reciprocity

Born's reciprocity program exchanges position and momentum roles. After a fixed conversion of units, its elementary linear exchange may be written schematically as

$$B_a(q,p) = \left(ap, -\frac{q}{a}\right), \qquad a \neq 0. \tag{5}$$

The sign and scale make this a canonical phase-space exchange rather than a mere swap of two scalars. With the canonical two-form

$$\sigma = dq \wedge dp, \tag{6}$$

one has $B_a^*\sigma = \sigma$. Born's wider program concerns spacetime and momentum-energy variables, invariant phase-space structures, and elementary particle theory. The historical proposal does not identify entropy with negative log coherence.

2.3 Hopf duality and the Majid comparison gate

Majid's duality principle develops an observable-state or position-momentum symmetry using Hopf algebras, quantum geometry, representations, and quantum Fourier transformation. At the algebraic core, a duality between Hopf algebras H and K may be expressed by a compatible pairing

$$\langle\ ,\ \rangle : H \times K \longrightarrow \mathbb{C} \tag{7}$$

obeying, schematically,

$$\begin{aligned} \langle ab, c\rangle &= \langle a \otimes b, \Delta_K c\rangle, \\ \langle a, cd\rangle &= \langle \Delta_H a, c \otimes d\rangle, \\ \langle 1_H, c\rangle &= \epsilon_K(c), \qquad \langle a, 1_K\rangle = \epsilon_H(a), \end{aligned} \tag{8}$$

with the corresponding antipode compatibility when the structures require it. Topological, braided, and infinite-dimensional settings require suitable refinements; equations (7)-(8) state only the algebraic comparison needed here.

For this paper, a **Majid-comparison datum** records the minimum fields that a positive FRC claim would have to supply:

$$\mathfrak{M} = (H, K, \langle\ ,\ \rangle, \Sigma, \mathsf{Rep}, \mathcal{I}, \mathcal{O}). \tag{9}$$

Here H, K are independently declared algebraic or geometric objects, $\langle\ ,\ \rangle$ is a compatible duality pairing, Σ is an exchange or semidualisation operation with a declared composition/equivalence rule, Rep records representation-level meaning, $\mathcal{I}$ records the preserved or exchanged structure, and $\mathcal{O}$ is the operational or physical interpretation. This is a comparison checklist distilled from the primary constructions, not a claim that every form of duality must use one identical tuple.

2.4 Semidualisation

In Majid and Schroers' three-dimensional quantum-gravity examples, semidualisation is an algebraic operation on quantum-group or matched-pair data. It exchanges position and momentum roles, relates quantum doubles and bicrossproduct-type models, acts on the associated representation theory, and can interchange physical scales. The statement that a model is self-dual is made at a specified algebraic level.

An involution of two derived real numbers does not supply that operation. FRC would need to identify the matched or dually paired structures and show that the operational entropy and coherence channels arise from them before the word *semidualisation* could be used as a result.

2.5 Definitions and evidential meaning

Term	Definition in this paper	Evidential meaning
Ledger datum	The tuple $\mathcal{L}$ in (3).	Exact coordinate geometry only.
State-space lift	A map $\widetilde{D} : X \to X'$ satisfying $\ell' \circ \widetilde{D} = D \circ \ell$.	Shows that the coordinate action is induced by underlying states or models.
Canonical lift	A state-space lift determined by independently supplied structure rather than chosen after observing D.	Candidate structural evidence.

Hopf pairing	A pairing compatible with product, coproduct, unit, counit, and the relevant antipodes.	Relates two algebraic structures as duals.
Tautological algebraization	An algebra action obtained automatically by pulling scalar functions back along D.	Algebraic syntax, not a physical duality.
Semidualisation	A declared operation exchanging paired algebraic sectors and their model roles.	Stronger than scalar exchange.
Majid-style self-duality	Equivalence with a declared dual at an algebraic/representational level, with composition and physical meaning stated.	Requires a completed datum such as (9).
Present-structure obstruction	Failure of the structures currently established by FRC to satisfy necessary comparison gates.	Rejects present promotion but permits a future independently constructed enrichment.

3. Exact comparison with the canonical Born exchange

The superficial resemblance between (3) and (5) disappears once composition and the canonical two-form are checked.

Proposition 1. The FRC involution is not the canonical Born exchange

For every $a \neq 0$, the canonical Born map B_a and the FRC ledger map D obey

$$B_a^2 = -I, \qquad B_a^4 = I, \qquad D^2 = I, \tag{10}$$

and

$$B_a^*(dq \wedge dp) = dq \wedge dp, \qquad D^*(dx \wedge dy) = -dx \wedge dy. \tag{11}$$

Their matrices have determinants

$$\det B_a = +1, \qquad \det D = -1. \tag{12}$$

Consequently B_a and D are not conjugate by any bijection, and therefore not by any real invertible linear, affine, smooth, or nonlinear change of coordinates.

Proof. Direct substitution into (5) gives

$$B_a^2(q,p) = B_a\left(ap, -\frac{q}{a}\right) = (-q,-p),$$

and hence $B_a^4 = I$. Equation (4) gives $D^2 = I$. Differentiating (5),

$$dq' = a\,dp, \qquad dp' = -\frac{1}{a}\,dq,$$

so

$$dq' \wedge dp' = a\,dp \wedge \left(-\frac{1}{a}dq\right) = dq \wedge dp.$$

For D, $dx' = -dy$ and $dy' = -dx$, hence

$$dx' \wedge dy' = dy \wedge dx = -dx \wedge dy.$$

The determinant statements follow from the matrices

$$[B_a] = \begin{pmatrix} 0 & a \\ -1/a & 0 \end{pmatrix}, \qquad [D] = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}. \tag{13}$$

The order already proves the strongest statement: conjugacy by a bijection h would give $(hB_ah^{-1})^2 = hB_a^2h^{-1} \neq I$, whereas $D^2 = I$. The determinant calculation independently excludes linear conjugacy and records the different orientation behavior. □

Proposition 1 does not say that every version of Born reciprocity must be the single map (5). It proves the bounded comparison relevant to the visual analogy usually made with the elementary position-momentum exchange. FRC's D has a different composition law and orientation behavior even before the absence of physical phase-space variables is considered.

The sign $\det D = -1$ is not evidence that D is antiunitary. Antiunitarity is a statement about conjugate-linear action on a complex Hilbert space and transition probabilities, not about the determinant of a two-coordinate real ledger map. The actual Wigner-class obstruction is stated separately in Section 6.

The form $dx \wedge dy$ in (11) is used only as the standard ambient area form on the ledger plane. FRC has not independently established it as a physical symplectic form. Equation

(11) says that, if one makes this purely geometric comparison, D reverses that form while the canonical Born exchange preserves its phase-space form.

4. The ledger does not reconstruct its underlying states

The decisive obstruction is not the sign in Proposition 1. It is the loss of information in passing from independently structured states to two ledger coordinates.

Theorem 1. State-lift non-reconstruction

The ledger datum $\mathcal{L} = (M_+, \omega, D)$ does not determine a unique state-space involution inducing D. In particular, there exist two inequivalent involutive lifts on the same state space with different fixed-state behavior and exactly the same ledger action.

Proof. First consider the identity presentation

$$X_1 = M_+, \qquad \ell_1 = \mathrm{id}_{M_+}, \qquad \widetilde{D}_1 = D. \tag{14}$$

It satisfies $\ell_1 \circ \widetilde{D}_1 = D \circ \ell_1$.

Now add a two-point fiber invisible to the ledger:

$$X_2 = M_+ \times \{0, 1\}, \qquad \ell_2(z, i) = z. \tag{15}$$

Define

$$\widetilde{D}_{2,0}(z, i) = (Dz, i), \qquad \widetilde{D}_{2,1}(z, i) = (Dz, 1 - i). \tag{16}$$

Because $D^2 = I$, both maps are involutions. Both make the same square commute:

$$\ell_2 \circ \widetilde{D}_{2,j} = D \circ \ell_2, \qquad j \in \{0, 1\}. \tag{17}$$

Their fixed-state behavior differs. If $z \in \mathrm{Fix}(D)$, then every (z, i) is fixed by $\widetilde{D}_{2,0}$. No (z, i) is fixed by $\widetilde{D}_{2,1}$, because $i = 1 - i$ has no solution in $\{0, 1\}$. Thus the two lifts cannot be conjugate by a bijection: conjugacy preserves the fixed set. Nevertheless their ledger images and induced coordinate action are identical. □

Corollary 1. The forgetful ledger map is non-injective

Forgetting the state-space fibers and retaining only (M_+, D) loses enough information to identify inequivalent structural actions. It therefore cannot reconstruct, without extra input, a canonical state map, observable algebra, duality pairing, or representation theory.

This is a non-reconstruction result, not a non-existence result. An independently defined experiment, algebra, dynamics, boundary, or universal property could select one lift. The theorem says that the ledger relation does not perform that selection.

The two-point construction is the minimal counterexample, not the only source of ambiguity. FRC 830.004, Theorem 4, gives a natural qubit-fiber version: the full qubit state space has continuous fibers over the same entropy and off-diagonal-coherence ledger point, and two inequivalent involutions in a declared fiber trivialization induce the same D. The non-reconstruction is therefore structural in an operational register actually used by FRC, rather than an artifact of appending an invisible bit. Those lifts are set-theoretic witnesses; they do not by themselves establish a physical quantum channel.

5. Why a function-algebra action is still insufficient

Every map of a space acts contravariantly on scalar functions. That fact is useful, but it must not be mistaken for observable-state duality.

Proposition 2. Minimal Hopf algebraization does not close the Majid gate

Let

$$A = \mathbb{R}[x, y] \tag{18}$$

be the polynomial coordinate algebra of the ambient ledger plane, with the standard primitive Hopf structure

$$\begin{aligned} \Delta x &= x \otimes 1 + 1 \otimes x, \qquad & \Delta y &= y \otimes 1 + 1 \otimes y, \\ \epsilon(x) &= \epsilon(y) = 0, & \mathcal{S}(x) &= -x, \quad \mathcal{S}(y) = -y. \end{aligned} \tag{19}$$

Pullback along D defines

$$D^\sharp : A \longrightarrow A, \qquad D^\sharp f = f \circ D, \qquad D^\sharp x = -y, \qquad D^\sharp y = -x. \tag{20}$$

Then $D^\sharp$ is an involutive Hopf-algebra automorphism of A. This fact does not define a Majid-style self-duality of an FRC operational register.

Proof. Pullback is multiplicative and unital:

$$D^\sharp(fg) = (D^\sharp f)(D^\sharp g), \qquad D^\sharp 1 = 1. \tag{21}$$

Since $D^2 = I$, $(D^\sharp)^2 = I$. On the generator x,

$$\Delta(D^\sharp x) = \Delta(-y) = -y \otimes 1 - 1 \otimes y = (D^\sharp \otimes D^\sharp)\Delta x, \tag{22}$$

and the same calculation holds for y. Unit, counit, and antipode compatibility follow on the generators and therefore on all of A. $\square$

Proposition 2 records the strongest elementary algebraization available without adding new physics. It also shows why saying merely "there is a Hopf algebra" is not enough. The primitive Hopf structure in (19) comes from the additive ambient coordinate plane; it would exist for any two linear coordinates and is not derived from entropy, coherence, a bath, a quantum state space, or an experimental operation. Moreover:

- $D^\sharp$ maps one chosen coordinate algebra to itself rather than pairing

two independently defined observable and state structures;

- no nondegenerate pairing (7) is supplied between operational FRC objects;
- no matched-pair or bicrossproduct data is identified;
- no quantum Fourier transform or semidual model is constructed;
- no representation is exchanged with a geometry; and
- the antipode of the ambient additive group does not preserve the operational

cone M_+.

The algebraization is valid mathematics. Its evidential level is P2 ledger geometry with an ambient algebra action, not P3/P4 structural or physical self-duality.

6. General and full-qubit lift boundaries

The non-reconstruction theorem says the ledger cannot choose a state action. FRC 830.004 asks the stronger existence question first for a general one-parameter trade and then for the full qubit state space.

Theorem 2. General one-parameter identity-lift obstruction

Let $t \mapsto (s(t), C(t))$ be a C^1 family on an interval, with $C(t) \in (0, 1]$ strictly monotone. Put $y = \ln C$ and suppose $ds/dy < 0$. If τ induces D on any subset of that family, then

$$\tau(t) = t, \qquad s(t) + y(t) = 0. \tag{23}$$

Thus every lift is the identity restricted to the zero set of $u = s + y$, and no nonidentity lift exists.

Proof. The coordinate y parametrizes the family and

$$\frac{dv}{dy} = \frac{ds}{dy} - 1 < -1, \qquad v = s - y. \tag{24}$$

Hence v is injective. Because D preserves v, a commuting lift satisfies $v(\tau(t)) = v(t)$ and therefore $\tau(t) = t$. Its u component then requires $u = -u$, hence $u = 0$. □

No discreteness conclusion follows from C^1 regularity alone. Excluding an open-domain identity lift additionally requires that $u^{-1}(0)$ have empty interior; analyticity or explicit shape information can establish that fact. For an exact normalized reciprocity curve,

$$dS_\mu + k_\mu^* d\ln C = 0 \quad \Longleftrightarrow \quad ds + dy = 0, \tag{25}$$

so $u = c$ on each connected component. If $c \neq 0$, there is no same-family lift; if $c = 0$, the unique induced lift on a ledger-injective family is the identity. This conclusion has no case split based on a numerical bridge representation.

The von Mises theorem of FRC 830.003 is an exact corollary with stronger root information. There v is injective and u is strictly unimodal with exactly two zeros, so D has no lift on a nonempty open concentration interval and no lift moving a von Mises density. Its valid Legendre duality between κ and r is not the FRC ledger exchange.

6.1 The qubit dimension threshold

For a qubit with Bloch radius R, off-diagonal coherence $C_{\text{off}} = \sqrt{r_x^2 + r_y^2}$ in a fixed basis, and $s = h((1+R)/2)$, FRC 830.004 proves the exact ledger image

$$\mathcal{R} = \{(s, y) : 0 \leq s < \ln 2,\ y \leq \ln R(s)\}, \qquad K = \mathcal{R} \cap D(\mathcal{R}). \tag{26}$$

The region K is nonempty and two-dimensional and contains a one-dimensional fixed leaf $u = 0$. This is the dimension threshold at which injectivity of one preserved coordinate no longer collapses a proposed lift to the identity. It creates room for a lift question; it does not answer that question.

Proposition 3. Scoped physical-lift obstructions

On the fixed-basis qubit ledger:

1. no incoherent operation realizes D on all of K, because D strictly

increases the declared coherence on the nonempty region $u < 0$;

1. no unital qubit CPTP channel realizes D on all of K, because D

strictly decreases entropy on the nonempty region $u > 0$; and

1. no unitary or antiunitary Wigner symmetry realizes D at any point off

$u = 0$, because both preserve the spectrum and hence entropy.

The third result is the **Wigner-class obstruction**. It derives from an actual unitary or conjugate-linear state action, not from the orientation of the ledger plane. These three hypotheses define different operation classes and cannot be combined into a no-CPTP theorem. A general nonunital, coherence-generating CPTP channel, a resource-assisted protocol, or an instrument-conditioned construction remains open and must be tested by a predeclared commuting square.

7. Gate-by-gate comparison

The structures can now be compared without relying on analogy.

Gate	Born / Majid-side requirement	Present FRC object	Verdict
Underlying sectors	Position/momentum, observables/states, or declared dual structures.	Entropy and log coherence are derived scalar channels; registers are typed but generally inequivalent.	Partial at ledger level only.
Exchange map	Canonical exchange, quantum Fourier duality, or semidualisation on the underlying sectors.	D acts exactly on (x, y).	Present only on the ledger.
Composition rule	Declared repeated action or equivalence.	$D^2 = I$.	Present at P2.

Canonical geometry	Phase-space/symplectic or explicitly declared quantum geometry.	$\omega = dx + dy$ is anti-invariant; the ambient area form is reversed.	Does not match canonical Born exchange.
Algebra	Algebra of coordinates/observables tied to the system.	Optional polynomial coordinate algebra A with $D^\sharp$.	Formal and tautological; no operational derivation.
Coalgebra	Coproduct encoding composition or symmetry.	Primitive ambient coproduct can be chosen.	Formal ambient choice; not an FRC physical result.
Compatible dual pairing	Pairing such as (7)-(8) between declared objects.	None supplied between FRC registers.	Absent.
Semidual data	Matched pair, bicrossproduct, or systematic replacement by a dual.	No matched pair or bicrossproduct attached to S and C.	Absent.
Representations	Representation theory with model or wave-equation meaning.	No representation action induced by D.	Absent.
State-space lift	Exchange induced by independently defined states/models.	Non-reconstructible in natural qubit fibers; impossible nontrivially under the one-parameter hypotheses; incoherent, unital, and Wigner classes fail on the stated qubit domains.	Several exact obstructions; general CPTP gate open.
Physical scales/invariants	Declared position/momentum scales, Casimirs, or exchanged constants.	k^* is the scale-invariant Boltzmann bridge represented numerically as k^*_μ in a declared register; it is not a semidualised dynamical scale or an independently evolving field.	Absent.
Empirical operation	Measurable implementation of the structural exchange.	None supplied.	Absent.

Theorem 3. Present Majid-gate obstruction

The structures established in FRC 830.000--830.004 do not form a Majid-comparison datum of the type (9), and they do not establish Born reciprocity.

Proof. Proposition 1 excludes identification of D with the elementary canonical Born exchange: the composition, determinant, and two-form behavior differ. Theorem 1 proves that the ledger datum does not determine a state-level exchange. Proposition 2 shows that even an involutive Hopf automorphism of the ambient coordinate algebra supplies neither an independently defined dual object nor the compatible pairing, semidualisation, or representation data in (9). Theorem 2 rejects every nonidentity one-parameter lift under its stated monotonicity hypotheses. FRC 830.004 makes the non-reconstruction natural in the qubit register and Proposition 3 excludes three major operation classes, while carefully leaving the general CPTP question open. At least the dual pairing, semidual operation, representation-level meaning, and demonstrated nontrivial operational lift gates are therefore absent or failed. The present structures cannot be promoted to the claimed class. □

The word *present* is essential. Theorem 3 is stable under new notation but not under genuinely new structure. If a later paper independently constructs the missing objects or a valid open-class lift, those objects must be tested by the reopening gate in Section 9 rather than ruled out by definition.

8. Relation to prior work

8.1 Born

Born's 1949 papers propose reciprocity between spacetime and momentum-energy variables in elementary-particle theory. Proposition 1 uses only the elementary canonical sign exchange, not the full historical theory. The comparison result is therefore narrow: the FRC reflection-like involution is not the same linear transformation as the canonical phase-space rotation. No claim is made that this observation evaluates Born's wider program.

8.2 Majid's duality principle

Majid's 1994 account explicitly frames the duality principle as a symmetry between observables and states enabled by Hopf algebras and quantum geometry. The dual Hopf structures, compatible pairings, and representation-theoretic interpretation do the mathematical work. His later account describes quantum Born reciprocity through Fourier duality and its non-Abelian/Hopf-algebraic generalisations.

FRC presently agrees with the methodological demand—name the structures and maps—but does not satisfy the construction. Its exact involution is on a two-dimensional ledger image. Section 4 proves why that image cannot recover the underlying action.

8.3 Majid and Schroers' semidualisation

The three-dimensional quantum-gravity analysis relates concrete quantum-group models by an algebraic semidualisation, follows their limits and scale exchanges, and studies their representations as wave equations. It also states the algebraic level at which a q-deformed model is self-dual. No analogous matched quantum-group data, representation family, or model exchange has been derived from FRC entropy and coherence.

8.4 Information geometry is a genuine but different duality

The von Mises exponential family has natural coordinate κ, expectation coordinate r, convex potential ψ, and Legendre dual $\psi^* = -S$. That duality is mathematically established. Theorem 2 shows that the different map $D(S, \ln r) = (-\ln r, -S)$ does not induce a nonidentity action on the same family. Rejecting the FRC lift preserves rather than weakens the valid information-geometric structure.

9. Claim-status table, kill result, and reopening gate

ID	Statement	Status	Basis
P1	The canonical Born exchange and FRC D have different order, determinant, and two-form behavior and are not conjugate by any bijection.	Proved proposition	Equations (10)-(13) and conjugacy invariance of order.
T1	The ledger datum does not reconstruct a unique state involution.	Proved theorem	Explicit two-fiber lifts (15)-(17).
C1	Ledger forgetting is non-injective on structural lifts.	Proved corollary	Distinct fixed-state behavior with identical ledger action.
P2	$D^\sharp$ is a Hopf automorphism of the standard primitive ambient coordinate algebra.	Proved formal proposition	Equations (18)-(22).
T2	Every lift on a monotone one-parameter entropy-coherence trade is the identity restricted to $u = 0$; the von Mises root count is a stronger corollary.	Inherited and reproduced theorem	Injectivity of v; FRC 830.003--830.004.

T2q	The full qubit ledger contains a two-dimensional D-invariant region and natural non-reconstructible fibers.	Inherited theorem	FRC 830.004, Theorems 3--4.
P3	Incoherent operations, unital qubit CPTP channels, and Wigner symmetries each fail as global/off-leaf lifts on their stated domains.	Inherited scoped no-go results	FRC 830.004, Theorem 5.
T3	Present FRC does not satisfy the Born/Majid comparison gates.	Proved present-structure obstruction	P1, T1, P2, T2, P3, and the missing data in Section 7.
R1	An ambient Hopf automorphism alone is a Majid-style self-duality.	Rejected	It lacks a dual operational object, pairing, semidualisation, representations, and physical lift.
R2	The point $\kappa r = 1$ is a quantum Born reciprocity point.	Rejected in the tested family	FRC 830.003 proves it is a maximum of $S + \ln r$, not a D-fixed or semidual point.
O1	A general nonunital, coherence-generating CPTP channel induces D on a nonempty qubit region.	Open	Requires one predeclared channel and commuting square.
O2	A different operational register admits a nontrivial structural lift of D.	Open	Requires an independent state/model map and commuting square.

9.1 Kill result

The paper's kill condition is met. Shared vocabulary, scalar exchange, $D^2 = I$, $D^*\omega = -\omega$, category syntax, or an automatically induced function-algebra automorphism do not establish Born reciprocity or Majid-style self-duality. The one-parameter theorem rejects nontrivial lifts under broad monotonicity hypotheses, and the first two-dimensional operational arena rejects three important operation classes while leaving the wider CPTP gate open. Present FRC is therefore **not promoted** beyond ledger geometry, typed operational morphisms, and scoped obstruction results.

This kill result does not erase the positive mathematics. FRC retains:

- the unique exact affine sign-reversing involution D on M_+ in the stated

class;

- the weak and strong register categories;
- an exact embedding of the von Mises family into the broader phase-density

register;

- the von Mises family's genuine Legendre geometry;
- a general one-parameter identity-lift obstruction and exact qubit dimension

threshold;

- natural qubit-fiber non-reconstruction; and
- three scoped physical-lift obstructions with the broader CPTP gate left

open.

9.2 Reopening gate

A future positive comparison must supply, before promotion:

1. independently defined algebraic, geometric, state, or observable objects;
2. the relevant product and coproduct or a comparably explicit structure;
3. a compatible and suitably nondegenerate duality pairing;
4. an exchange, Fourier, or semidualisation map not fitted from the desired

ledger identity;

1. a composition/equivalence or self-duality rule;
2. a nontrivial state/model lift inducing the ledger action;
3. representation-level or dynamical consequences; and
4. an operational interpretation with units and boundaries declared.

FRC 830.004 supplies the dimension threshold and several negative operation- class results, but it does not supply a Hopf pairing or semidualisation. A future driven-dissipative or general CPTP construction would contribute only to the lift gate. Each gate remains separate.

10. What this paper does not establish

This paper does not establish:

- that no future FRC algebra, coalgebra, pairing, or duality can exist;
- a classification of all nonlinear lifts or cross-register maps;
- a new Hopf algebra, bicrossproduct, braided group, or quantum Fourier

transform;

- a refutation of Born reciprocity or Majid's program;
- a position-momentum interpretation of entropy and coherence;
- a quantum-gravity model;
- a physical state-observable exchange;
- a no-go theorem for every CPTP channel or resource-assisted operation;
- a running k^* coupling, beta function, scale connection, or independent

bridge dynamics;

- a universal law for $dS + k^* d \ln C = 0$; or
- empirical evidence for FRC self-duality.

The conclusion is narrower and exact: the structures currently established by FRC do not meet the comparison gates, and the ledger data alone cannot reconstruct the missing action.

Appendix A. Exact reproducibility record

No fit, simulation, or empirical data enters the new results.

For Proposition 1, matrix multiplication gives

$$\begin{pmatrix} 0 & a \\ -1/a & 0 \end{pmatrix}^2 = -I, \qquad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^2 = I, \tag{30}$$

and the determinants are $+1$ and -1.

For Theorem 1, the three commuting checks are

$$\begin{aligned} \ell_1(\widetilde{D}_1 z) &= Dz, \\ \ell_2(\widetilde{D}_{2,0}(z,i)) &= Dz, \\ \ell_2(\widetilde{D}_{2,1}(z,i)) &= Dz. \end{aligned} \tag{31}$$

Squaring each lift gives the relevant identity map. At a fixed ledger $z = Dz$, the fixed-set conditions are respectively unrestricted i and the impossible equation $i = 1 - i$.

For Proposition 2, checking the algebra, coproduct, counit, and antipode on the two polynomial generators suffices because A is generated by x, y and all maps involved are algebra morphisms or antimorphisms of the declared type.

For Theorem 2, the decisive calculation on any permitted one-parameter family is

$$v(\tau(t)) = v(t), \qquad \frac{dv}{dy} = \frac{ds}{dy} - 1 < -1 \quad \Longrightarrow \quad \tau(t) = t. \tag{32}$$

The von Mises root count and domain checks are reproduced fully in FRC 830.003. The qubit image, fixed-leaf endpoint, natural fiber lifts, and three operation-class proofs are reproduced in FRC 830.004 and its deterministic verification script.

References

1. H. Servat, *FRC 830.000 v0.1 - What Would Make FRC Reciprocity a Duality?* (2026).
2. H. Servat, *FRC 830.001 v0.1 - The Reciprocity One-Form* (2026).
3. H. Servat, *FRC 830.002 v0.1 - Operational Registers and Reciprocity-Preserving Morphisms* (2026).
4. H. Servat, *FRC 830.003 v0.1 - Exact Phase-Family Test of FRC Duality* (2026).
5. H. Servat, *FRC 830.004 v0.1 - Dimension Threshold and Physical-Lift Obstructions* (2026).
6. H. Servat, *FRC 566.001 v2.2 - Entropy-Coherence Reciprocity and the Universal Coherence Condition* (2026).
7. M. Born, "Elementary Particles and the Principle of Reciprocity," *Nature* **163**, 207-208 (1949), [doi:10.1038/163207a0](https://doi.org/10.1038/163207a0).
8. M. Born, "Reciprocity Theory of Elementary Particles," *Reviews of Modern Physics* **21**, 463-473 (1949), [doi:10.1103/RevModPhys.21.463](https://doi.org/10.1103/RevModPhys.21.463).
9. S. Majid, "Principle of Representation-Theoretic Self-Duality," *Physics Essays* **4**(3), 395-405 (1991), [doi:10.4006/1.3028923](https://doi.org/10.4006/1.3028923).
10. S. Majid, "Duality Principle and Braided Geometry" (1994), [arXiv:hep-th/9409057](https://arxiv.org/abs/hep-th/9409057), related DOI [10.1007/3-540-59163-X_265](https://doi.org/10.1007/3-540-59163-X_265).
11. S. Majid and B. J. Schroers, "q-Deformation and Semidualisation in 3d Quantum Gravity" (2008; revised 2009), [arXiv:0806.2587](https://arxiv.org/abs/0806.2587).
12. S. Majid, "On the Emergence of the Structure of Physics," *Philosophical Transactions of the Royal Society A* **376**, 20170231 (2018), [doi:10.1098/rsta.2017.0231](https://doi.org/10.1098/rsta.2017.0231), [arXiv:1711.00556](https://arxiv.org/abs/1711.00556).
13. S.-I. Amari, "Differential Geometry of Curved Exponential Families—Curvatures and Information Loss," *The Annals of Statistics* **10**, 357-385 (1982), [doi:10.1214/aos/1176345779](https://doi.org/10.1214/aos/1176345779).

14. T. Baumgratz, M. Cramer, and M. B. Plenio, "Quantifying Coherence," *Physical Review Letters* **113**, 140401 (2014), [arXiv:1311.0275](https://arxiv.org/abs/1311.0275).
15. R. Simon, N. Mukunda, S. Chaturvedi, and V. Srinivasan, "Two Elementary Proofs of the Wigner Theorem on Symmetry in Quantum Mechanics" (2008), [arXiv:0808.0779](https://arxiv.org/abs/0808.0779).