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Dimension Threshold for FRC Reciprocity v0.1: One-Parameter and Qubit Lift Obstructions

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Abstract

> *The FRC ledger involution D(s,y)=(-y,-s), with normalized entropy s=S_mu/k*_mu and y=ln C, cannot act nontrivially on any one-parameter family whose coherence is strictly monotone and whose normalized entropy decreases with log coherence. The preserved coordinate v=s-y is then injective, so every lift is the identity restricted to u=s+y=0. Exact normalized reciprocity ds+dy=0 sharpens the result: a connected curve lies on u=c; if c is nonzero there is no same-family lift, while c=0 permits only the trivial identity on a ledger-injective family. The wrapped-Cauchy family supplies an exact Poisson-kernel example with stationary coherence 1/sqrt(3) and*

> *algebraic fixed ledgers. The full qubit state space crosses the dimension threshold: its two-dimensional ledger image contains a nonempty D-invariant region K with a one-dimensional fixed leaf and admits explicit inequivalent set-theoretic fiber lifts. Those lifts are not physical promotions. No incoherent operation, no unital qubit CPTP channel, and no unitary or antiunitary Wigner symmetry realizes D on all of K. A general nonunital coherence-generating CPTP or resource-assisted lift remains open.*

0. Result in one paragraph

The one-parameter obstruction of FRC 830.003 is not specific to the von Mises family. In any C^1 one-parameter family with strictly monotone coherence and $ds/d\ln C < 0$, the coordinate $v = s - \ln C$ is injective. Because the FRC involution D preserves v, every proposed lift fixes the family parameter and survives only where $u = s + \ln C = 0$. Thus no nonidentity lift exists. Exact normalized reciprocity makes the exclusion sharper: $ds + d\ln C = 0$ places a connected family on one leaf $u = c$, which D sends to $u = -c$; only the leaf $c = 0$ survives, and there the induced map is the identity when the ledger is injective. A wrapped-Cauchy family proves the result without Bessel functions and gives algebraic fixed ledgers. The full qubit ledger then shows what changes in two dimensions. Its image contains a nonempty D-invariant region K and explicit inequivalent state-fiber involutions inducing the same ledger action. That is a set-theoretic existence and non-reconstruction result, not a physical duality. Incoherent operations, unital qubit channels, and Wigner unitary/antiunitary symmetries are separately excluded on all of K. The general nonunital, coherence-generating CPTP and resource-assisted questions remain open.

1. Question, scope, and dependency record

FRC 830.000--830.003 classified the normalized ledger involution, typed its operational lifts, and proved that it cannot move a fixed-mean von Mises density. Two questions remained mixed together:

1. Is that obstruction a special property of one statistical family?
2. If a ledger image has more than one dimension, does removal of the

one-parameter obstruction produce a physical lift?

This paper answers both questions with bounded results. The first obstruction is general under an explicit monotone-trade hypothesis. The qubit ledger then crosses the dimension threshold and admits nonunique set-theoretic lifts, but three declared physical operation classes fail. Dimension two is therefore necessary for escaping the injective-v proof in the studied setting; it is not sufficient for physical duality.

This paper addresses the exact lift question because its mathematical objects and kill conditions can be stated before evaluation. It does not claim that stochastic-pump geometry is absent.

Source	Result used here	Status used here
FRC 566.001 v2.2	Scale-invariant bridge and normalized operational law.	Controlling notation.
FRC 830.000 v0.1	Claim classes and ledger involution.	Exact framework candidate.
FRC 830.001 v0.1	Adapted coordinates and affine classification.	Exact algebra candidate.
FRC 830.002 v0.1	Strong lift as a commuting square on states and ledgers.	Exact categorical candidate.
FRC 830.003 v0.1	Von Mises injective-v obstruction.	Exact family theorem candidate.
FRC 830.005 v0.1	Ledger non-reconstruction and Born/Majid comparison gate.	Provisional dependency to be revised after this result.

Every theorem below is pending independent proof review. Deterministic scripts verify numerical brackets and contract language; they are not independent mathematical review or empirical evidence.

2. Normalized ledger and lift definitions

2.1 Scale-invariant coordinates

Declare an operational register μ. The canonical k^* is the scale-invariant Boltzmann bridge, while k^*_μ is its numerical representation in the declared entropy unit. Define

$$s = \frac{S_\mu}{k^*_\mu}, \qquad y = \ln C, \qquad \ell = (s, y). \tag{1}$$

A lawful entropy-unit conversion multiplies S_μ and k^*_μ by the same positive factor and leaves C unchanged, so both coordinates in (1) are unchanged. For numerical examples this paper declares the natural-information representation μ_{nat} with $k^*_{\mu_{\text{nat}}} = 1$ only after the invariant formulas have been stated.

On the normalized ledger plane, define

$$D(s, y) = (-y, -s), \qquad u = s + y, \qquad v = s - y. \tag{2}$$

Then

$$D(u, v) = (-u, v), \qquad D^2 = I. \tag{3}$$

There is no theorem-level distinction between numerical representations in which k^*_μ has different positive values. Equation (1) makes that representation independence explicit.

2.2 Lifts and nontriviality

Let X be a declared state or model space with ledger map $\ell : X \to \mathbb{R}^2$. A lift of D on $E \subseteq X$ is a predeclared map $\tau : E \to X$ satisfying

$$\ell \circ \tau = D \circ \ell. \tag{4}$$

A nonidentity lift moves at least one underlying state. A set-theoretic lift need not be continuous, affine, CPTP, operational, or canonical. A physical claim must separately declare and verify the relevant operation class.

3. General one-parameter obstruction

Theorem 1. General one-parameter identity-lift obstruction

Let I be an interval and let

$$t \longmapsto \big(s(t), C(t)\big)$$

be a C^1 family with $C(t) \in (0, 1]$ strictly monotone. Put $y(t) = \ln C(t)$ and assume

$$\frac{ds}{dy} < 0 \tag{5}$$

throughout the domain. If $E \subseteq I$ and $\tau : E \to I$ satisfies

$$\ell(\tau(t)) = D\ell(t), \tag{6}$$

then

$$\tau(t) = t, \qquad u(t) = 0 \tag{7}$$

for every $t \in E$.

Equivalently, every lift is the identity restricted to the zero set of u. No nonidentity lift exists under these hypotheses.

Proof. Because y is a valid family coordinate,

$$\frac{dv}{dy} = \frac{ds}{dy} - 1 < -1. \tag{8}$$

Thus v is strictly decreasing in y, hence injective along the family. Equations (3) and (6) give

$$v(\tau(t)) = v(t).$$

Injectivity forces $\tau(t) = t$. The u component of (6) then gives $u(t) = -u(t)$, hence $u(t) = 0$. ☐

Corollary 1. Zero-set qualification

No discreteness conclusion follows from C^1 regularity alone. Theorem 1 excludes an open-domain lift only when $u^{-1}(0)$ has empty interior. If u is real analytic and not identically zero, its roots are isolated. If u is strictly unimodal, has negative endpoint limits, and has a positive maximum, it has exactly two roots.

Reason. The C^1 function u may vanish on an interval. In particular, the exact relation $ds = -dy$ with integration constant zero gives precisely such an interval. Analyticity or the stated shape hypotheses supply the extra root information; Theorem 1 alone does not. ☐

Theorem 2. Exact reciprocity excludes nontrivial one-parameter duality

Under the hypotheses of Theorem 1, suppose a connected family exactly obeys

$$dS_\mu + k_\mu^* \, d \ln C = 0. \tag{9}$$

Then $u = s + y = c$ is constant. If $c \neq 0$, no same-family D lift exists. If $c = 0$, D fixes the full ledger curve and the unique induced map on a ledger-injective family is the identity.

Proof. Divide (9) by the declared register representation k_μ^* to get

$$ds + dy = 0. \tag{10}$$

Thus $du = 0$ and connectedness gives $u = c$. Equation (3) maps the leaf $u = c$ to $u = -c$. For $c \neq 0$ these leaves are disjoint, so the image cannot lie in the same family. For $c = 0$, every ledger point is fixed. Ledger injectivity then forces the state map to be the identity. □

Theorem 2 is the normalized reciprocity--nontrivial-duality exclusion. It has no case split based on the numerical value of the bridge representation. It does not cover hidden fibers, higher-dimensional images, nonmonotone trades, or cross-register maps.

4. Exact wrapped-Cauchy example

The next example shows that Theorem 1 is not a consequence of the special Bessel-function identities used in FRC 830.003.

Proposition 1. Wrapped-Cauchy harmonic ledger

For $0 < \rho < 1$, let

$$p_\rho(\theta) = \frac{1}{2\pi} \frac{1 - \rho^2}{1 + \rho^2 - 2\rho \cos\theta}, \qquad 0 \leq \theta < 2\pi. \tag{11}$$

In the declared natural-information representation, the mean-resultant coherence and differential entropy are

$$C(\rho) = \rho, \qquad s(\rho) = \ln\bigl(2\pi(1 - \rho^2)\bigr). \tag{12}$$

Proof. The density in (11) is the Poisson kernel for the disk point $w = \rho$. If H is harmonic on a neighborhood of the closed unit disk, the Poisson formula gives

$$\int_0^{2\pi} p_\rho(\theta) H(e^{i\theta}) \, d\theta = H(\rho). \tag{13}$$

Applying (13) to $H(w) = \operatorname{Re} w$ gives $\mathbb{E}_\rho[\cos\theta] = \rho$, while symmetry gives zero sine moment. Hence the mean-resultant magnitude is $C = \rho$.

For the entropy, write

$$\ln p_\rho(\theta) = \ln \frac{1-\rho^2}{2\pi} - \ln|1-\rho e^{i\theta}|^2. \tag{14}$$

The function $H_\rho(w) = \ln|1-\rho w|^2$ is harmonic on $|w| < 1/\rho$, a domain containing the closed unit disk. Equation (13) gives

$$\mathbb{E}_\rho \ln|1-\rho e^{i\theta}|^2 = H_\rho(\rho) = 2\ln(1-\rho^2). \tag{15}$$

Substitution into $s = -\mathbb{E}_\rho \ln p_\rho$ proves (12). □

Corollary 2. Algebraic fixed ledgers and family-dependent stationarity

For the family (11),

$$u(\rho) = \ln\bigl(2\pi(1-\rho^2)\rho\bigr), \qquad v(\rho) = \ln\frac{2\pi(1-\rho^2)}{\rho}. \tag{16}$$

Their derivatives are

$$v'(\rho) = -\frac{2\rho}{1-\rho^2} - \frac{1}{\rho} < 0, \tag{17}$$

and

$$u'(\rho) = \frac{1}{\rho} - \frac{2\rho}{1-\rho^2} = 0 \quad\Longleftrightarrow\quad \rho_* = \frac{1}{\sqrt{3}}. \tag{18}$$

The fixed-ledger equation $u = 0$ is the cubic

$$\rho^3 - \rho + \frac{1}{2\pi} = 0. \tag{19}$$

It has exactly two roots in $(0, 1)$:

$$\rho_- = 0.163527904172\ldots, \qquad \rho_+ = 0.908157240947\ldots. \tag{20}$$

Deterministic sign brackets are

$$u(0.1635279) < 0 < u(0.1635280), \qquad u(0.9081572) > 0 > u(0.9081573). \tag{21}$$

Proof. Equations (16)--(18) follow from (12). The derivative u' is positive below $1/\sqrt{3}$ and negative above it. Moreover $u \to -\infty$ at both endpoints and $u(1/\sqrt{3}) > 0$, so there is exactly one root on each side of the maximum. Exponentiating $u = 0$ yields (19). The sign brackets locate the roots without supplying their existence proof. ☐

The wrapped-Cauchy stationary coherence $1/\sqrt{3} \approx 0.57735$ differs from the von Mises value $0.62178\ldots$. Stationary location is a property of the chosen family. The invariant conclusion is the injective-v obstruction, not a universal numerical threshold.

The wrapped-Cauchy/Poisson-kernel family is the phase-density form associated with the Ott--Antonsen invariant manifold in standard globally coupled oscillator models. That dynamical relevance does not turn (16) into an FRC state-space duality.

5. The full qubit ledger

5.1 State space and image

Fix a computational basis and write

$$\rho = \frac{1}{2}\big(I + \mathbf{r}\cdot\boldsymbol{\sigma}\big), \qquad \mathbf{r} = (r_x, r_y, r_z), \qquad R = |\mathbf{r}| \le 1. \tag{22}$$

Use the declared coherence

$$C_{\mathrm{off}} = 2|\rho_{01}| = \sqrt{r_x^2 + r_y^2}, \tag{23}$$

and von Neumann entropy in nats,

$$s = S_{\mathrm{vN}}(\rho) = h\left(\frac{1+R}{2}\right), \tag{24}$$

where

$$h(p) = -p \ln p - (1-p)\ln(1-p). \tag{25}$$

Let $R(s)$ denote the unique inverse of (24) on $0 \le s < \ln 2$, chosen so $R(0) = 1$ and $R(s) \downarrow 0$.

Theorem 3. Exact qubit ledger image and dimension threshold

For $C_{\text{off}} > 0$, the image of the qubit state space under $\ell(\rho) = (s, \ln C_{\text{off}})$ is

$$\mathcal{R} = \{(s, y) : 0 \le s < \ln 2,\ y \le \ln R(s)\}. \tag{26}$$

The maximal D-invariant subregion is

$$K = \mathcal{R} \cap D(\mathcal{R}), \tag{27}$$

or equivalently

$$\begin{aligned} 0 \le s < \ln 2, \qquad & -\ln 2 < y \le 0, \\ e^{y} \le R(s), \qquad & e^{-s} \le R(-y). \end{aligned} \tag{28}$$

The set K is nonempty and two-dimensional. Its D-fixed set is the segment

$$\operatorname{Fix}(D) \cap K = \{(s, -s) : 0 \le s \le s_\dagger\}, \tag{29}$$

where

$$R_\dagger = 0.610973770564\ldots, \qquad s_\dagger = -\ln R_\dagger = 0.492701249431\ldots. \tag{30}$$

Proof. Positivity of (22) is $R \le 1$. At fixed entropy, (24) fixes R. Equation (23) gives $0 < C_{\text{off}} \le R$, so every qubit ledger lies in (26). Conversely, for any point in (26), choose

$$\mathbf{r} = \left(e^{y}, 0, \sqrt{R(s)^2 - e^{2y}}\right). \tag{31}$$

It is a valid Bloch vector with the required entropy and coherence, proving the reverse inclusion.

Equations (27)--(28) follow by requiring both (s, y) and $D(s, y) = (-y, -s)$ to satisfy (26). Invariance is immediate: $D(K) = D(\mathcal{R}) \cap \mathcal{R} = K$.

On the fixed line $y = -s$, membership reduces to $e^{-s} \le R(s)$. Parameterize by R and define

$$f(R) = h\left(\frac{1+R}{2}\right) + \ln R. \tag{32}$$

Then

$$f'(R) = \frac{1}{R} - \operatorname{artanh} R, \qquad f''(R) = -\frac{1}{R^2} - \frac{1}{1-R^2} < 0. \tag{33}$$

Thus f' decreases from $+\infty$ to $-\infty$ and f has one maximum. Also $f(R) \to -\infty$ as $R \downarrow 0$, $f(1) = 0$, and the explicit interior witness

$$f(0.9) = 0.0931547276\ldots > 0 \tag{34}$$

proves that the maximum is positive. Hence there is one interior root $R_\dagger$ in addition to the boundary root $R = 1$, and $f \ge 0$ precisely between them. This proves (29). The bracket

$$f(0.6109737) < 0 < f(0.6109738) \tag{35}$$

locates the interior endpoint. Finally, $(0.3, -0.3)$ satisfies both inequalities in (28) strictly, so an open neighborhood lies in K. Therefore K is two-dimensional. $\square$

The one-dimensional proof of Theorem 1 fails on K because a level set of $v = s - y$ is now a line segment rather than a single state-family point. That is the dimension threshold. It removes one obstruction; it supplies no preferred state action.

6. Natural qubit-fiber non-reconstruction

Let K°_{fib} be the part of K for which both a ledger point and its D image satisfy the strict inequalities

$$0 < e^y < R(s). \tag{36}$$

Each ledger fiber there has the representation

$$\mathbf{r}(s, y, \phi, \varepsilon) = \left(e^y \cos\phi, e^y \sin\phi, \varepsilon\sqrt{R(s)^2 - e^{2y}}\right), \tag{37}$$

where $\phi \in \mathbb{R}/2\pi\mathbb{Z}$ and $\varepsilon \in \{+1, -1\}$.

Theorem 4. Natural qubit-fiber non-reconstruction

In the declared fiber trivialization (37), define

$$\widetilde{D}_0(s, y, \phi, \varepsilon) = \big(D(s, y), \phi, \varepsilon\big), \tag{38}$$

and

$$\widetilde{D}_1(s, y, \phi, \varepsilon) = \big(D(s, y), \phi + \pi, \varepsilon\big). \tag{39}$$

Both are involutive set-theoretic state maps on the represented strict interior, and both induce D on the ledger. They are inequivalent: over the fixed ledger leaf, $\widetilde{D}_0$ fixes every represented state while $\widetilde{D}_1$ fixes none.

Proof. The definition of K°_{fib} ensures that (37) is valid at both the source and target ledger. Applying either map twice returns the ledger because $D^2 = I$. It returns ϕ exactly for $\widetilde{D}_0$ and modulo 2π for $\widetilde{D}_1$. Both preserve the displayed fiber label and therefore square to the identity. Forgetting (ϕ, ε) gives $D(s, y)$ in both cases.

If $y = -s$, then the source and target ledgers coincide. Equation (38) leaves all four displayed coordinates unchanged. Equation (39) would require $\phi = \phi + \pi$ modulo 2π, which has no solution. Conjugate maps have bijections between their fixed sets, so the two lifts are inequivalent. □

Theorem 4 replaces an artificial hidden bit with fibers already present in a declared operational register. It still does not select a canonical lift. The trivialization, azimuth, and sign label were chosen before defining the two maps, but other choices exist. The ledger forgets exactly the information that distinguishes them.

7. Physical operation-class obstructions

For $(s, y) \in K$, the ledger target is

$$s' = -y, \qquad C' = e^{-s}. \tag{40}$$

Therefore

$$u = s + y < 0 \Longrightarrow C' > C, \qquad u = s + y > 0 \Longrightarrow s' < s. \tag{41}$$

Both strict regions are nonempty: $(0.2, -0.3) \in K$ has $u < 0$, and its D image $(0.3, -0.2) \in K$ has $u > 0$.

Theorem 5. Operation-class lift obstructions

For the fixed-basis qubit ledger above:

1. no incoherent operation realizes D on all of K;
2. no unital qubit CPTP channel realizes D on all of K; and
3. no unitary or antiunitary Wigner symmetry realizes D on any set

containing a point off the fixed leaf $u = 0$.

Proof.

1. For a qubit, the l_1 coherence in the declared basis is

$C_{l_1} = 2|\rho_{01}| = C_{\text{off}}$. It is nonincreasing under the declared class of incoherent operations. But (41) requires a strict increase on the nonempty $u < 0$ part of K. One operation cannot induce D on all of K.

1. A unital qubit channel acts affinely on the Bloch ball with zero

translation, $\mathbf{r} \mapsto T\mathbf{r}$. Positivity maps the unit ball into itself, so the operator norm of T is at most one and $|T\mathbf{r}| \leq |\mathbf{r}|$. The entropy (24) decreases with Bloch radius; consequently a unital channel cannot decrease s. Equation (41) demands a strict entropy decrease on the nonempty $u > 0$ part of K.

1. A unitary state symmetry has the form

$\rho \mapsto U\rho U^\dagger$. An antiunitary Wigner symmetry induces $\rho \mapsto U\rho^T U^\dagger$ after a basis is chosen. Both preserve the spectrum of ρ and hence preserve s. A D lift instead requires $s' = -y$. Equality with the original entropy holds only when $s = -y$, or $u = 0$. Thus neither Wigner class induces D off the fixed leaf. □

These are three different no-go results. The first uses a resource-theoretic coherence monotone, the second uses unitality, and the third uses spectral preservation. They cannot be combined into a no-CPTP theorem. A general nonunital, coherence-generating CPTP channel belongs to none of the excluded classes in parts 1--3. Whether one fixed such channel, a resource-assisted protocol, or an instrument-conditioned map can induce D on a nonempty qubit region remains open.

Wigner's theorem begins with a map of quantum rays preserving transition probabilities and concludes unitary or antiunitary implementability. The orientation reversal of the derived ledger plane is not itself a Wigner hypothesis. Theorem 5 tests Wigner-class state maps only after declaring the qubit state space, and it finds an entropy obstruction rather than a candidate time-reversal realization.

8. Relation to FRC 830.003 and FRC 830.005

FRC 830.003 proved more family-specific information than Theorem 1: its von Mises u is strictly unimodal, has exactly two roots, and has its maximum at $\kappa r = 1$. Theorem 1 extracts the part that generalizes—the injectivity of v and consequent absence of a nonidentity lift. Proposition 1 supplies a second exact family and shows why the location of the maximum does not generalize.

FRC 830.005 proved that the ledger cannot reconstruct a unique state action and that a tautological Hopf algebraization does not supply Majid-style self-duality. Theorem 4 makes the non-reconstruction operationally natural in the qubit register. Theorem 5 then closes three tempting physical routes while leaving the broader channel question open. None of these results supplies a Hopf pairing, semidualisation, representation exchange, or physical FRC self-duality.

The logical sequence is now:

$$\text{one parameter} \Longrightarrow \text{no nonidentity lift}; \qquad \text{two-dimensional qubit ledger} \Longrightarrow \text{no}$$

The arrow in the second statement is not a promotion. It is the point at which the lift question becomes nontrivial and must be answered operation class by operation class.

9. Claim-status table

ID	Statement	Status	Basis
T1	Every lift on a monotone one-parameter trade is the identity restricted to $u = 0$.	Proved candidate	Injectivity of v.
C1	Root discreteness requires an additional analytic or shape hypothesis.	Proved qualification	Explicit $u = 0$ interval countercase.
T2	Exact normalized reciprocity excludes nontrivial same-family duality on a ledger-injective one-parameter family.	Proved candidate	Constant-u leaf classification.
P1	Wrapped Cauchy has harmonic ledger formulas, stationary $C = 1/\sqrt{3}$, and two algebraic fixed ledgers.	Proved candidate	Poisson formula and cubic equation.

T3	The qubit ledger image contains a nonempty two-dimensional D-invariant region K with a fixed leaf.	Proved candidate	Bloch-ball image and binary entropy.
T4	Two inequivalent qubit-fiber involutions induce the same D.	Proved set-theoretic candidate	Explicit maps (38)--(39).
T5a	No incoherent operation induces D on all of K.	Proved within declared resource theory	Coherence increase on $u < 0$.
T5b	No unital qubit channel induces D on all of K.	Proved candidate	Entropy decrease on $u > 0$.
T5c	No unitary/antiunitary Wigner symmetry induces D off $u = 0$.	Proved candidate	Spectral entropy preservation.
O1	A general nonunital coherence-generating CPTP channel induces D on a nonempty region.	Open	Requires one predeclared channel and commuting square.
O2	A resource-assisted or instrument-conditioned map realizes D.	Open	Resource and conditioning record absent.

10. Reopening gate

A future positive physical-lift claim must provide before evaluation:

1. a fixed state domain and ledger image;
2. one predeclared state map or channel rather than a fiberwise output choice;
3. complete positivity and trace preservation if the claim is a deterministic

quantum channel;

1. any coherence resource, ancilla, measurement outcome, or postselection

used by the implementation;

1. a commuting-square proof on a nonempty region containing non-fixed ledger

points;

1. a composition rule and operational consequence beyond the two scalar

outputs; and

1. an explanation of how the construction relates, or does not relate, to the

algebraic gates of FRC 830.005.

Passing this gate would establish a structural lift in one register. It would not automatically establish Born reciprocity, Majid-style self-duality, or a universal FRC law.

11. What this paper does not establish

This paper does not establish:

- that every one-parameter family has a monotone entropy--coherence trade;
- that C^1 zero sets are isolated;
- that dimension two is sufficient for a physical lift;
- a canonical qubit-fiber lift;
- a general no-CPTP theorem;
- a stochastic connection, curvature, pump, or holonomy;
- a time-reversal implementation of D;
- a physical symplectic form on the ledger plane;
- a running coupling, beta function, scale connection, or independent

dynamics for k^*;

- a Hopf algebra derived from the qubit ledger;
- a Born-, Majid-, or representation-theoretic self-duality; or
- empirical support from a quantum processor.

The exact result is narrower: the one-parameter nonidentity lift is obstructed; the two-dimensional qubit ledger permits nonunique set-theoretic lifts; three physical operation classes fail; and a broader driven-dissipative channel gate remains open.

Appendix A. Deterministic reproducibility

The tracked command is

```
npm run test:830:004
```

The script `scripts/verify-frc-830-004.mjs` computes:

- the two sign-changing roots of (19) by bisection;
- the inverse binary-entropy radius $R(s)$;
- the unique interior root of (32);

- strict-interior witnesses $(0.2, -0.3)$ and $(0.3, -0.2)$; and
- the exact involution check $D^2 = I$ on those witnesses.

Binary64 values locate roots after the analytic existence and uniqueness proofs. No random numbers, fit, grid-classification claim, or adaptive model choice enters the result.

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