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Operational Registers and Reciprocity-Preserving Morphisms v0.1

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Abstract

> *FRC uses several legitimate coherence routes, including phase order, the von Mises mean resultant, quantum purity, basis-dependent off-diagonal interference, and platform-specific composites. This paper formalizes how those routes may share a framework without being declared the same observable. A weak reciprocity map preserves the pulled-back ledger one-form up to a nonzero multiplier. A strong affine register morphism additionally supplies a state or model map and a commuting ledger square. Both classes compose; the strong class forms a category with explicit identities, associativity, exact-preserving and signed subcategories, and an invertible groupoid. One-form preservation alone is shown insufficient by an exact counterexample. The von Mises family supplies a nontrivial strong morphism into the phase-distribution register: the state-space inclusion commutes exactly with the entropy and mean-resultant ledger, but is not an isomorphism. A qubit no-go theorem proves that purity and fixed-basis off-diagonal coherence admit no single-valued ledger adapter under the identity state map, even though both are valid operational routes. Present FRC registers therefore form a typed formal category populated by a sparse graph of verified arrows, not one proven physical equivalence class.*

0. Result in one paragraph

FRC's coherence routes can inhabit one mathematical framework without being one observable. The framework is a category of typed register presentations: a strong arrow contains both a map on state or model spaces and a compatible map on the entropy-coherence ledger. Identity and composition are proved, not assumed. The category is populated sparsely. The von Mises family has an exact state-space embedding into the broader phase-distribution route. In contrast, quantum purity and fixed-basis off-diagonal coherence cannot be identified by any state-preserving ledger map, because distinct pure qubit states have the same entropy and purity but different off-diagonal coherence. This is not a ledger trick: the positive example acts on distributions before it acts on ledger coordinates, and the negative example prevents a coordinate adapter from erasing physically different meanings.

1. Question, scope, and status

At a declared operational register, FRC writes

$$dS_\mu + k^*_\mu d\ln C_\mu = 0, \qquad k^*_\mu > 0, \quad C_\mu \in (0,1]. \tag{1}$$

The subscript records a representation, state space, observable, unit convention, and boundary. It does not make all objects carrying the symbol C_μ identical.

This distinction matters because the current corpus uses several established routes:

- a Kuramoto or circular mean-resultant phase order;
- the same mean resultant restricted to the von Mises family;
- purity of a density matrix or noise-averaged ensemble;
- a basis-dependent off-diagonal interference measure; and
- platform-specific composites used for transport or prediction.

A framework is allowed to use all of these. Science advances by citing and combining established objects. The burden is not to reinvent them. The burden is to state which state spaces and observables they belong to, when a map between them exists, and what that map preserves.

This paper supplies:

1. a weak category based on one-form pullback;
2. a stronger category based on a commuting state/ledger square;
3. an exact state-space embedding for the von Mises route;
4. an exact no-go theorem for silently identifying purity with off-diagonal

coherence under the identity state map; and

1. an operational acceptance record for future adapters.

The category theorems are formal results. They do not prove that every formal arrow is physical. The two route results are exact calculations under their declared domains and remain subject to independent review.

2. Typed operational registers

Define a typed operational register

$$\mathcal{R}_\mu = (X_\mu, S_\mu, C_\mu, k^*_\mu, B_\mu), \tag{2}$$

where:

- X_μ is a state, parameter, trajectory, or model space;
- $S_\mu : X_\mu \to \mathbb{R}$ is the declared entropy channel;
- $C_\mu : X_\mu \to (0, 1]$ is the declared coherence channel;
- $k^*_\mu > 0$ is the predeclared unit bridge; and
- B_μ records the type, basis where relevant, boundary, environment,

sampling procedure, units, and admissible domain.

Its ledger map and pulled-back reciprocity form are

$$\ell_\mu : X_\mu \longrightarrow M_\mu, \qquad \ell_\mu(z) = \left(\frac{S_\mu(z)}{k^*_\mu}, \ln C_\mu(z)\right), \qquad \Omega_\mu = \ell^*_\mu(dx + dy). \tag{3}$$

The form Ω_μ lives on the state or model space. It is therefore different from the ambient ledger form $\omega_\mu = dx + dy$.

2.1 Definitions

Term	Definition	Evidential meaning
Register	The typed tuple (2), not merely a number called coherence.	Declares what is measured and where.
Ledger map	The map ℓ_μ in (3).	Converts independently defined channels to normalized coordinates.
Weak reciprocity map	A state/model map preserving Ω up to a declared nonzero multiplier.	Preserves the total differential form but may hide channel rearrangement.
Strong affine adapter	A state/model map plus an affine ledger map making a commuting square.	Shows how state-space and coordinate actions agree.
Isomorphism	A strong adapter possessing a strong inverse.	Formal equivalence of typed presentations, not automatically physical identity.
Embedding	An injective strong adapter not required to be onto.	Places one route inside a broader route without equating them.
Empirical realization	A strong adapter whose state map, units, boundary, and observables are independently implemented.	Physical evidence beyond the formal category.

2.2 Required operational adapter record

Before an arrow is scored as operational rather than purely formal, it must declare:

Field	Required content

Source and target	Exact register IDs and state/model spaces.
State/model map	A predeclared function $f : X_\mu \to X_\nu$.
Ledger map	A predeclared map $A : M_\mu \to M_\nu$.
Multiplier	The fixed $\lambda_A \neq 0$ in $A^*\omega_\nu = \lambda_A\omega_\mu$.
Domains	Endpoint, positivity, smoothness, and exclusion conditions.
Units and Boltzmann bridge	Register, units, and the joint numerical conversion rule for S_μ and k^*_μ declared before outcomes.
Boundary and basis	System/environment split and any measurement basis.
Invertibility	Whether the arrow is an embedding, quotient, channel, or isomorphism.
Baseline	Equal-information alternative without the claimed adapter.
Evidence status	Definition, proof, simulation, measurement, or conjecture.
Kill condition	The observation or contradiction that rejects the arrow.

An analogy or a fitted curve is not an adapter record.

2.3 Unit-equivalent registers

Two presentations $\mathcal{R}_\mu$ and $\mathcal{R}_\nu$ of the same operational register are **unit equivalent** when they have the same state space, coherence observable, and boundary data up to a declared change of entropy units, and there is a predeclared $a_{\nu\mu} > 0$ such that

$$S_\nu = a_{\nu\mu} S_\mu, \qquad k^*_\nu = a_{\nu\mu} k^*_\mu, \qquad C_\nu = C_\mu.$$

Unit conversions compose according to

$$a_{\rho\nu} a_{\nu\mu} = a_{\rho\mu}, \qquad a_{\mu\mu} = 1, \qquad a_{\mu\nu} = a_{\nu\mu}^{-1}.$$

The canonical k^* is the scale-invariant Boltzmann bridge, and k^*_μ is its numerical register representation. Equation (2) is a representation change, not a new law, a fitted coefficient, or an independently evolving state variable. Changing the coherence channel, observable, state space, or boundary is not a unit equivalence and requires a separate declared adapter.

3. Weak reciprocity maps

Theorem 1. The weak category

Let objects be pairs (X_μ, Ω_μ) with a differentiable state/model space and a one-form. A weak morphism from μ to ν is a pair

$$(f, \lambda) : (X_\mu, \Omega_\mu) \longrightarrow (X_\nu, \Omega_\nu), \qquad f^*\Omega_\nu = \lambda\Omega_\mu, \quad \lambda \in \mathbb{R}^\times. \tag{4}$$

These objects and arrows form a category $\mathsf{WeakRec}$.

Proof. The identity arrow is

$$(\mathrm{id}_{X_\mu}, 1), \qquad \mathrm{id}_{X_\mu}^* \Omega_\mu = \Omega_\mu. \tag{5}$$

If $(f, \lambda) : \mu \to \nu$ and $(g, \eta) : \nu \to \rho$, define

$$(g, \eta) \circ (f, \lambda) = (g \circ f, \eta\lambda). \tag{6}$$

Pullback functoriality gives

$$(g \circ f)^* \Omega_\rho = f^*(g^* \Omega_\rho) = f^*(\eta \Omega_\nu) = \eta\lambda\Omega_\mu. \tag{7}$$

Thus composition is closed. Associativity follows from associativity of function composition and multiplication in $\mathbb{R}^\times$. The identity laws follow from (5). □

This category is mathematically valid but deliberately weak. It sees only the total pulled-back form. It need not preserve the individual meaning of the entropy and coherence coordinates.

4. Strong ledger-lifted adapters

Let $\mathcal{G}_{\mathsf{aff}}$ denote the affine reciprocity group classified in FRC 830.001. Its elements satisfy

$$A^* \omega_\nu = \lambda_A \omega_\mu, \qquad \lambda_A \neq 0, \tag{8}$$

with declared source and target domains.

A strong affine morphism from $\mathcal{R}_\mu$ to $\mathcal{R}_\nu$ is a pair (f, A) such that

$$\ell_\nu \circ f = A \circ \ell_\mu. \tag{9}$$

Equation (9) is the commuting-square condition. It requires the state/model map to induce the declared coordinate action; neither map may be inferred after viewing the desired residual.

Theorem 2. The strong category and its grading

Typed register presentations and strong affine morphisms form a category $\mathsf{LedReg}_{\mathsf{aff}}$. Every strong morphism induces a weak morphism. The invertible strong arrows form a groupoid.

Proof. The identity on $\mathcal{R}_\mu$ is

$$(\mathrm{id}_{X_\mu}, I), \qquad \ell_\mu \circ \mathrm{id}_{X_\mu} = I \circ \ell_\mu. \tag{10}$$

For $(f, A) : \mu \to \nu$ and $(g, B) : \nu \to \rho$, define

$$(g, B) \circ (f, A) = (g \circ f, B \circ A). \tag{11}$$

Then

$$\ell_\rho \circ (g \circ f) = B \circ \ell_\nu \circ f = B \circ A \circ \ell_\mu, \tag{12}$$

so the square commutes. FRC 830.001 proves closure and associativity of the affine ledger maps; function composition is associative. Equation (10) supplies both identity laws. Hence the strong objects and arrows form a category.

For the weak implication, use (3), (8), and (9):

$$\begin{aligned} f^*\Omega_\nu &= f^*\ell_\nu^*\omega_\nu = (\ell_\nu \circ f)^*\omega_\nu \\ &= (A \circ \ell_\mu)^*\omega_\nu = \ell_\mu^* A^*\omega_\nu = \lambda_A \Omega_\mu. \end{aligned} \tag{13}$$

If both f and A are invertible and their inverses satisfy the reverse commuting square, (f^{-1}, A^{-1}) is a strong inverse. Such arrows contain identities and are closed under composition and inverse, so they form a groupoid. □

Corollary 1. Exact and signed subcategories

Under composition, multipliers obey

$$\lambda_{B\circ A} = \lambda_B \lambda_A. \tag{14}$$

Therefore:

- arrows with $\lambda = 1$ form an exact-preserving subcategory;
- arrows with $\lambda \in \{1, -1\}$ form a $\mathbb{Z}_2$-graded subcategory;
- reversing arrows with $\lambda = -1$ alone do not form a subcategory, because

they contain no identity and two reversals compose to $\lambda = 1$; and

- two registers are formally equivalent only when they are isomorphic in the

strong category.

Corollary 2. Unit-equivalent presentations are strongly isomorphic

Every lawful unit equivalence from $\mathcal{R}_\mu$ to $\mathcal{R}_\nu$ defines the exact strong isomorphism

$$(\mathrm{id}_X, I) : \mathcal{R}_\mu \longrightarrow \mathcal{R}_\nu, \qquad \ell_\nu \circ \mathrm{id}_X = I \circ \ell_\mu.$$

These isomorphisms compose according to the conversion-factor cocycle.

Proof. The unit-equivalence equations give

$$\frac{S_\nu}{k_\nu^*} = \frac{a_{\nu\mu} S_\mu}{a_{\nu\mu} k_\mu^*} = \frac{S_\mu}{k_\mu^*}, \qquad \ln C_\nu = \ln C_\mu.$$

Hence $\ell_\nu = \ell_\mu$, so the state identity and ledger identity satisfy the commuting square with multiplier 1. The inverse conversion gives the strong inverse, and the cocycle law gives composition. □

Shared notation, equal dimensions, or equal numerical coherence values do not define an isomorphism.

5. Counterexample: weak preservation need not lift affinely

Counterexample 1. Equal total forms, different channel geometry

Let $X = [0, \infty)$ and define two ledger maps

$$\ell_1(t) = (t, 0), \qquad \ell_2(t) = (t + t^2, -t^2). \tag{15}$$

Both lie in M_+: their first coordinates are nonnegative and their second coordinates are nonpositive. Their pulled-back forms are

$$\Omega_1 = d(t) = dt, \qquad \Omega_2 = d\big((t + t^2) - t^2\big) = dt. \tag{16}$$

Thus $(\mathrm{id}_X, 1)$ is an exact weak morphism. Suppose an affine ledger map A satisfied

$$\ell_2 = A \circ \ell_1. \tag{17}$$

The affine image $A(t, 0)$ is affine in t in each coordinate. The right side of (17) would therefore have zero second derivative, while both components of ℓ_2 contain a nonzero quadratic term. This is impossible.

The total one-form can agree while entropy and log coherence are rearranged in a way that the declared affine ledger class does not lift. A weak match is therefore insufficient evidence for a strong register relation.

6. Exact phase-family lift

6.1 The phase-distribution register

Let X_ϕ be the smooth strictly positive probability densities on the circle whose first circular moment is nonzero. In the natural-information register μ_{nat}, with $k^*_{\mu_{\mathrm{nat}}} = 1$, define

$$H[p] = -\int_0^{2\pi} p(\theta) \ln p(\theta)\, d\theta, \qquad C_\phi[p] = \left| \int_0^{2\pi} e^{i\theta} p(\theta)\, d\theta \right| > 0, \tag{18}$$

and

$$\ell_\phi[p] = (H[p], \ln C_\phi[p]). \tag{19}$$

The angle coordinate and reference measure $d\theta$ are part of the register type. Because this is differential entropy, $H[p]$ need not be nonnegative; the ledger therefore uses the half-plane $M = \mathbb{R} \times (-\infty, 0]$, not necessarily the smaller cone M_+ of FRC 830.001.

This is the continuum version of the Kuramoto mean-resultant route. It is not a new coherence definition invented by FRC.

6.2 The von Mises register

Fix mean direction zero, let $X_{\mathrm{vM}} = (0, \infty)$, and define the state-space map

$$i : X_{\mathrm{vM}} \longrightarrow X_\phi, \qquad i(\kappa) = p_\kappa(\theta) = \frac{e^{\kappa\cos\theta}}{2\pi I_0(\kappa)}. \tag{20}$$

The restriction $\kappa > 0$ ensures $C_\phi > 0$ so that $\ln C_\phi$ is finite. The exact formulas are

$$C_{\mathrm{vM}}(\kappa) = \frac{I_1(\kappa)}{I_0(\kappa)} = r(\kappa), \qquad S_{\mathrm{vM}}(\kappa) = \ln\big(2\pi I_0(\kappa)\big) - \kappa r(\kappa). \tag{21}$$

Define

$$\ell_{\mathrm{vM}}(\kappa) = \big(S_{\mathrm{vM}}(\kappa), \ln C_{\mathrm{vM}}(\kappa)\big). \tag{22}$$

Theorem 3. The von Mises inclusion is a strong embedding

The pair (i, I) is a strong exact-preserving morphism

$$(i, I) : \mathcal{R}_{\mathrm{vM}} \longrightarrow \mathcal{R}_\phi, \tag{23}$$

because

$$\ell_\phi \circ i = \ell_{\mathrm{vM}} = I \circ \ell_{\mathrm{vM}}. \tag{24}$$

The identity ledger map also obeys $I^*\omega = \omega$, so the multiplier is $\lambda_I = 1$. It is an embedding, not an isomorphism.

Proof. Substituting p_κ into (18) gives exactly the two formulas in (21), proving the commuting relation (24). If $p_{\kappa_1} = p_{\kappa_2}$, then the difference of their log densities is

$$(\kappa_1 - \kappa_2)\cos\theta - \ln\frac{I_0(\kappa_1)}{I_0(\kappa_2)}, \tag{25}$$

which can vanish for all θ only when $\kappa_1 = \kappa_2$. Thus i is injective.

To show it is not onto, choose real a, b with $0 < |a| + |b| < 1$ and $ab \neq 0$, and define

$$p_{a,b}(\theta) = \frac{1 + a\cos\theta + b\cos 2\theta}{2\pi}. \tag{26}$$

The density is strictly positive because its numerator is at least $1 - |a| - |b| > 0$. Its first circular moment has magnitude $|a|/2 > 0$, so $p_{a,b} \in X_\phi$. A von Mises density with $\kappa > 0$ has nonzero Fourier coefficients $I_n(\kappa)/I_0(\kappa)$ for every $n \geq 1$, whereas (26) has zero coefficients for all $n \geq 3$. Hence $p_{a,b}$ is not von Mises. The map i is not surjective and cannot be an isomorphism. ☐

This is the first nontrivial state-space lift in the 830 series. It uses Kuramoto/von Mises mathematics exactly where that mathematics applies. Reusing a successful community result is not a loss; claiming that its subfamily is every phase distribution would be.

7. Quantum routes: an exact non-identification theorem

FRC 100.002 uses density-matrix purity as a primary ensemble-coherence channel and an off-diagonal channel as a secondary interference instrument. Those routes answer different questions. Purity is basis independent and measures mixedness; off-diagonal coherence is basis dependent and measures interference relative to a declared basis.

Fix the qubit basis $\{|0\rangle, |1\rangle\}$ and the common state domain

$$X_* = \{\rho \geq 0 : \operatorname{Tr}\rho = 1,\ 0 < 2|\rho_{01}| \leq 1\}. \tag{27}$$

In information units define

$$S_{\mathrm{vN}}(\rho) = -\operatorname{Tr}(\rho \ln \rho), \qquad C_{\mathrm{pur}}(\rho) = \operatorname{Tr}\rho^2, \qquad C_{\mathrm{off}}(\rho) = 2|\rho_{01}|, \tag{28}$$

with ledger maps

$$\ell_{\mathrm{pur}}(\rho) = \big(S_{\mathrm{vN}}(\rho), \ln C_{\mathrm{pur}}(\rho)\big), \qquad \ell_{\mathrm{off}}(\rho) = \big(S_{\mathrm{vN}}(\rho), \ln C_{\mathrm{off}}(\rho)\big). \tag{29}$$

Theorem 4. No identity-state ledger adapter from purity to off-diagonal coherence

There is no single-valued map A, affine or nonlinear, such that

$$\ell_{\mathrm{off}} = A \circ \ell_{\mathrm{pur}} \tag{30}$$

on all of X_* under the identity state map.

Proof. For $0 < \theta < \pi/2$, define

$$|\psi_\theta\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle, \qquad \rho_\theta = |\psi_\theta\rangle\langle\psi_\theta|. \tag{31}$$

Every ρ_θ is pure, so

$$S_{\mathrm{vN}}(\rho_\theta) = 0, \qquad C_{\mathrm{pur}}(\rho_\theta) = 1, \qquad C_{\mathrm{off}}(\rho_\theta) = \sin 2\theta. \tag{32}$$

At $\theta_1 = \pi/12$ and $\theta_2 = \pi/4$,

$$\ell_{\mathrm{pur}}(\rho_{\theta_1}) = \ell_{\mathrm{pur}}(\rho_{\theta_2}) = (0, 0), \tag{33}$$

but

$$\ell_{\mathrm{off}}(\rho_{\theta_1}) = \big(0, \ln(1/2)\big), \qquad \ell_{\mathrm{off}}(\rho_{\theta_2}) = (0, 0). \tag{34}$$

If (30) held, the single input $A(0, 0)$ would have to equal two different outputs. Contradiction. □

The theorem rejects a silent, state-preserving identification of the two routes. It does not prove that no quantum channel, basis change, coarse-graining, restricted state family, or independently calibrated experimental adapter can relate them. Any such proposal needs its own f, A, domain, and interface record.

8. Current route graph

Route	State/model space	Coherence functional	Essential type data	Verified arrows		
Phase/Kuramoto	Circle distributions or declared phase ensembles	$	\langle e^{i\theta}\rangle	$	Angle convention, sampling measure, ensemble construction	Receives the exact von Mises embedding.
von Mises	$\kappa > 0$ statistical family	$I_1(\kappa)/I_0(\kappa)$	Mean direction fixed at zero, reference measure, information-unit normalization, and family restriction	Exact strong embedding (i, I) into the phase route; not an isomorphism.		
Quantum purity	Density matrices or declared ensemble averages	$\mathrm{Tr}\,\rho^2$	Hilbert space, averaging/noise procedure, state normalization	Identity-state conversion to off-diagonal route rejected by Theorem 4.		
Off-diagonal interference	Density matrices with a fixed measurement basis and positive channel	Normalized off-diagonal magnitude	Basis, normalization, selected matrix band or full norm	Identity-state conversion from purity rejected; other adapters open.		
Transport composite	Platform-specific state/trajectory space	Predeclared composite of phase, spectrum, defects, or transport	Units, weights, window, boundary, equal-feature baseline	No universal cross-route arrow established.		

The present mathematical object is a formal category of typed presentations populated by a sparse graph of verified arrows. It is not one proven isomorphism class. Identity arrows exist for every declared route; the exact von Mises embedding is one nonidentity arrow; one tempting quantum identity arrow is ruled out; most cross-route arrows remain open.

9. Operational admission gate

A candidate strong arrow is admitted operationally only if:

1. f and A are declared before the target result is inspected;
2. S_μ and C_μ remain independently meaningful observables;
3. the commuting square is tested, not imposed by defining the target channel;
4. source and target units, domains, boundaries, and bases are compatible;
5. composition with already accepted arrows preserves their interface records;
6. an equal-information baseline is scored; and
7. the arrow is killed if it needs outcome-dependent fitting or collapses

physically distinct states that the target route distinguishes.

Theorem 3 passes this gate formally because the state inclusion i is defined independently and the ledger equality follows from the distribution. Theorem 4 fails the proposed identity adapter before fitting begins.

10. Relation to prior work

10.1 Category theory

Eilenberg and Mac Lane introduced categories, functors, and natural equivalences to formalize structure-preserving relationships. This paper uses only the elementary category axioms: objects, identity arrows, composable arrows, and associativity. It does not claim a new categorical foundation or invoke natural transformations, adjunctions, or higher categories.

10.2 Circular statistics and synchronization

The mean resultant in (18) is the standard circular-statistics object and the continuum form of the Kuramoto order parameter. Theorem 3 does not rename it as an FRC invention. The FRC contribution is the typed embedding and the rule that evidence from the von Mises subfamily may not be silently transferred to all phase distributions.

10.3 Quantum coherence

Resource-theoretic quantum coherence is basis dependent and has explicit axioms for valid measures. The normalized qubit off-diagonal channel in (28) is the qubit specialization of the standard l_1-norm route. Purity is a different, basis-independent mixedness quantity. Theorem 4 is therefore a type check made exact by two states, not a criticism of either standard measure.

10.4 Onsager and Majid

Onsager reciprocity concerns symmetry of linear-response coefficients under microscopic reversibility, not the category defined here. Majid's duality and semidualisation work uses declared algebraic structures and representation theory. The present category borrows that structural discipline—name the objects and arrows—but does not claim a Hopf-algebraic, representation- theoretic, or quantum-gravity duality.

11. Claim-status table and kill result

ID	Statement	Status	Basis

D1	Equations (2)-(3) define typed registers and ledger maps.	Definition	Formal-operational convention.
U1	Unit-equivalent presentations are exact strong isomorphisms with the identity normalized-ledger map.	Proved corollary	Joint transformation of S_μ and k_μ^* plus the conversion-factor cocycle.
T1	Weak reciprocity maps form $\mathsf{WeakRec}$.	Proved theorem	Identities, composition, and associativity in (5)-(7).
T2	Strong affine adapters form $\mathsf{LedReg}_{\mathrm{aff}}$ and induce weak maps.	Proved theorem	Commuting-square calculation (10)-(13).
C1	Exact arrows form a subcategory; signed arrows are $\mathbb{Z}_2$ graded; invertible arrows form a groupoid.	Proved corollary	Multiplier law (14) and inverse definition.
N1	Weak equality of pulled-back forms need not admit an affine ledger lift.	Exact counterexample	Equations (15)-(17).
T3	The von Mises inclusion is an exact strong embedding into the phase-distribution route.	Proved route theorem	Distribution-level map and exact ledger equality (20)-(26).
T4	No identity-state ledger map converts purity to off-diagonal coherence on all X_*.	Proved no-go theorem	Two pure qubit states in (31)-(34).
O1	A nonidentity physical adapter links purity and off-diagonal coherence on a restricted experiment.	Open problem	Requires a predeclared channel, basis, domain, and baseline.
O2	All current FRC routes belong to one isomorphism class.	Not established and contradicted for the identity purity/off-diagonal proposal	Theorem 4 and missing inverses.
O3	The involution D of FRC 830.001 lifts to the von Mises state family.	Open problem	Assigned to FRC 830.003.

The paper resolves its kill condition route by route:

- accept (i, I) because it has an independent state-space map and a commuting

ledger square;

- reject the purity-to-off-diagonal identity arrow because it is not even a

single-valued ledger map;

- reject any future outcome-fitted or meaning-erasing arrow; and
- retain multiple coherence routes as typed objects rather than forcing them

into one observable.

This is a valid compositional structure with a nontrivial populated arrow and a nontrivial obstruction. It neither collapses the routes nor abandons their shared framework.

12. What this paper does not establish

This paper does not establish:

- that the formal category is itself physical evidence;
- that every pair of registers has a morphism;
- that all coherence routes are equivalent or interchangeable;
- that the von Mises family represents every phase distribution;
- that purity is a basis-dependent interference measure;
- that off-diagonal coherence determines purity;
- that no physical channel can ever connect the two quantum routes;
- that a μ-register label proves cross-register causation;
- that the canonical FRC relation is a universal physical law; or
- that $D(x, y) = (-y, -x)$ lifts to distributions or quantum states.

FRC 830.003 now has a sharply typed task: test whether D induces a map on the von Mises family or whether Theorem 3's embedding exposes a lift obstruction. The answer must be obtained on probability distributions and their parameters, not only on ledger coordinates.

Appendix A. Exact reproducibility record

No fit or simulation enters the new results.

For Theorem 1, the only identities are

$$\mathrm{id}^* \Omega = \Omega, \qquad (g \circ f)^* = f^* g^*. \tag{35}$$

For Theorem 2, substitution of two commuting squares gives

$$\ell_\rho \circ g \circ f = B \circ \ell_\nu \circ f = B \circ A \circ \ell_\mu, \tag{36}$$

and pullback of the square gives (13).

For Counterexample 1,

$$\frac{d}{dt}\left[(t + t^2) + (-t^2)\right] = 1, \tag{37}$$

while the coordinate second derivatives are 2 and -2, excluding an affine image of $(t, 0)$.

For Theorem 3, positivity of (26) follows from

$$1 + a\cos\theta + b\cos 2\theta \geq 1 - |a| - |b| > 0, \tag{38}$$

and its absent harmonics $n \geq 3$ exclude equality with any $\kappa > 0$ von Mises density.

For Theorem 4, the pure-state density matrix is

$$\rho_\theta = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}. \tag{39}$$

It obeys $\rho_\theta^2 = \rho_\theta$ and $2|(\rho_\theta)_{01}| = \sin 2\theta$, reproducing (32)-(34).

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