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The Reciprocity One-Form v0.1: Affine Classification and Domain Rigidity

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Abstract

This paper classifies every invertible affine transformation of the normalized FRC ledger plane whose derivative preserves the reciprocity distribution ker(dx+dy). In coordinates u=x+y and v=x-y, the complete class is F(u,v)=(lambda u+c, alpha u+beta v+delta), with lambda beta nonzero. The paper derives the group law, inverse, leaf action, all affine involutions, and their fixed sets. Intersecting the classification with the operational half-plane M=R x (-infinity,0] yields a three-parameter family and a one-parameter family of sign-reversing involutions. Intersecting it with the nonnegative-entropy domain M+=[0,infinity) x (-infinity,0] is rigid: every kernel-preserving affine automorphism is either aI or aD, where a>0 and

> *D(x,y)=(-y,-x). Consequently the identity is the unique exact form-preserving automorphism, D is the unique exact form-reversing automorphism, and D is the unique nonidentity involution in this bounded class. The result is an exact coordinate theorem pending independent proof review. It does not lift D to physical states, observables, distributions, dynamics, or operational registers and therefore does not establish a physical, categorical, Born, or Majid-style duality.*

0. Result in one paragraph

The elementary affine symmetry problem is completely solvable. On the ambient ledger plane, every invertible affine map preserving the reciprocity kernel has five parameters and acts triangularly in the coordinates $u = x + y$ and $v = x - y$. Its composition, inverse, involutions, fixed sets, and leaf action are explicit. Requiring compatibility with $C \in (0, 1]$ sharply reduces the group. Requiring in addition $S/k^* \geq 0$ makes it rigid: the only surviving maps are positive multiples of the identity and positive multiples of $D(x, y) = (-y, -x)$. Thus D is the unique nonidentity affine involution of that domain preserving the reciprocity kernel. This is a positive coordinate-level result, not yet a physical duality. Independent proof review is still required before the result is labeled `exact-mathematical` in the evidence hierarchy.

1. Question, scope, and status

At one declared information-unit representation, with its register subscript suppressed, the canonical FRC relation is

$$dS + k^* d\ln C = 0, \qquad k^* > 0, \quad C \in (0, 1]. \tag{1}$$

FRC 830.000 showed that the map $D(x, y) = (-y, -x)$ reverses the sign of the normalized reciprocity one-form on a restricted domain. It did not establish whether D is isolated, part of a larger transformation group, or unique under meaningful domain constraints.

This paper answers those questions for one bounded class:

> *invertible affine self-maps of the ledger plane and of two declared ledger domains.*

The bounded class is deliberate. It is broad enough to include translations, linear scalings, shears, coordinate exchanges, reflections, and the candidate D, but narrow enough to admit a complete theorem. Nonlinear maps and non-invertible maps are exhibited later as counterexamples to any unbounded interpretation of the result.

The paper proves four theorems:

1. the complete ambient affine classification;
2. the complete classification of involutions in that group;
3. the classification on the operational half-plane $C \in (0, 1]$; and
4. the rigidity classification when normalized entropy is also nonnegative.

These are formal proofs with internal algebra checks. They remain candidates for `exact-mathematical` evidence until independently reviewed. Nothing here promotes the canonical FRC relation from operational bookkeeping to a universal physical law.

2. Ledger geometry and six preservation notions

Define

$$x = \frac{S}{k^*}, \qquad y = \ln C, qquad q(x, y) = x + y. \tag{2}$$

The three domains used in this paper are

$$\widetilde{M} = \mathbb{R}^2, qquad M = \mathbb{R} \times (-\infty, 0], \qquad M_+ = [0, \infty) \times (-\infty, 0]. \tag{3}$$

Here M enforces only $C \in (0, 1]$. The smaller domain M_+ additionally enforces $S/k^* \geq 0$. That extra condition is appropriate for many discrete and thermodynamic entropy conventions but not for every differential-entropy or offset convention. The two domains must therefore be analyzed separately.

The reciprocity one-form and its kernel are

$$\omega = dq = dx + dy, qquad \ker \omega = \{(\xi, \eta) : \xi + \eta = 0\}. \tag{4}$$

Introduce adapted coordinates

$$u = x + y, qquad v = x - y, qquad x = \frac{u + v}{2}, \qquad y = \frac{u - v}{2}. \tag{5}$$

Then $q = u$ and $\omega = du$. The u direction records the ledger total; the v direction runs within a level set of that total.

Let

$$F(z) = Az + b, qquad A \in GL(2, \mathbb{R}), \quad b \in \mathbb{R}^2 \tag{6}$$

be an invertible affine map. The following notions are distinct.

Notion	Exact condition	Meaning
Form preservation	$F^*\omega = \omega$	The normalized covector is unchanged.
Form reversal	$F^*\omega = -\omega$	The covector changes sign.
Constant conformal preservation	$F^*\omega = \lambda\omega$, $\lambda \neq 0$	The covector is rescaled by one nonzero constant.
Kernel preservation	$A(\ker \omega) = \ker \omega$	Tangent directions satisfying reciprocity remain such directions.
Zero-leaf preservation	$F(q^{-1}(0)) = q^{-1}(0)$	The particular ledger leaf $q = 0$ is mapped to itself.
Domain automorphism	$F(N) = N$ bijectively for declared N	Operational inequalities and endpoints are preserved.

Translations do not affect a pullback of a constant one-form, but they can move the level sets of q. In x, y coordinates, with $e = (1, 1)^T$, one has

$$F^*\omega = (A^T e)^T dz, qquad q(F(z)) = e^T Az + e^T b. \tag{7}$$

Thus preservation of a derivative object and preservation of a particular ledger value must not be conflated.

3. The ambient affine classification

Theorem 1. Complete affine kernel-preserving class

For an invertible affine map $F : \widetilde{M} \to \widetilde{M}$, the following are equivalent:

1. $A(\ker \omega) = \ker \omega$;
2. $F^*\omega = \lambda\omega$ for a unique $\lambda \neq 0$;

3. in the adapted coordinates (5), F has the form

$$F_{\lambda,c,\alpha,\beta,\delta}(u,v) = \big(\lambda u + c, \alpha u + \beta v + \delta\big), \qquad \lambda\beta \neq 0. \tag{8}$$

For every such map,

$$F^*\omega = \lambda\omega, \qquad q \circ F = \lambda q + c. \tag{9}$$

Proof. Suppose first that $A(\ker \omega) = \ker \omega$. The covector $A^T e$ annihilates every vector in $\ker e^T$. Since A is invertible, $A^T e \neq 0$. Two nonzero covectors on a two-dimensional vector space with the same one-dimensional kernel are proportional. Hence $A^T e = \lambda e$ for a unique $\lambda \neq 0$, which is equivalent to $F^*\omega = \lambda\omega$ by (7).

Conversely, $A^T e = \lambda e$ implies $e^T A v = \lambda e^T v = 0$ whenever $v \in \ker e^T$. Invertibility gives equality of the one-dimensional kernels, so $A(\ker \omega) = \ker \omega$.

In adapted coordinates, $F^* du = \lambda du$ means that the first coordinate has derivative $(\lambda, 0)$, hence $u' = \lambda u + c$. The most general second affine coordinate is $v' = \alpha u + \beta v + \delta$. Its Jacobian determinant is $\lambda\beta$, so invertibility is exactly $\lambda\beta \neq 0$. Equations (9) follow immediately. □

Corollary 1. Form, leaf, and zero-leaf subclasses

Let $L_s = q^{-1}(s)$. Theorem 1 gives

$$F(L_s) = L_{\lambda s + c}. \tag{10}$$

Therefore:

- exact form preservation is the subclass $\lambda = 1$;
- exact form reversal is the subclass $\lambda = -1$;
- the zero leaf is preserved exactly when $c = 0$; and
- every leaf is preserved individually exactly when $\lambda = 1$ and $c = 0$.

A form-preserving translation with $c \neq 0$ preserves the tangent distribution while permuting the leaves. This is the simplest reason not to identify $F^*\omega = \omega$ with preservation of $q = 0$.

4. Group structure

Write a map (8) as the parameter tuple $(\lambda, c, \alpha, \beta, \delta)$. If F_2 is applied after F_1, direct substitution gives

$$F_2 \circ F_1 = \big(\lambda_2\lambda_1, \lambda_2 c_1 + c_2, \alpha_2\lambda_1 + \beta_2\alpha_1, \\ \beta_2\beta_1, \alpha_2 c_1 + \beta_2\delta_1 + \delta_2\big). \tag{11}$$

The identity is

$$I = (1, 0, 0, 1, 0), \tag{12}$$

and the inverse is

$$F^{-1} = \left(\frac{1}{\lambda}, -\frac{c}{\lambda}, -\frac{\alpha}{\lambda\beta}, \frac{1}{\beta}, \frac{\alpha c}{\lambda\beta} - \frac{\delta}{\beta}\right). \tag{13}$$

Equations (11)-(13) prove closure, identity, and inverse explicitly. Associativity is inherited from composition of functions. Thus the ambient kernel-preserving affine maps form a group. The multiplier $F \mapsto \lambda$ is a homomorphism to $\mathbb{R}^\times$; exact form preservation is its $\lambda = 1$ subgroup, while form reversal is a coset rather than a subgroup because two reversals compose to a preservation.

5. Complete classification of affine involutions

Theorem 2. Involution conditions

A map (8) satisfies $F^2 = I$ if and only if

$$\lambda^2 = 1, qquad \beta^2 = 1, qquad (\lambda + 1)c = 0, qquad \alpha(\lambda + \beta) = 0, qquad \alpha c + (\beta + 1)\delta$$

Equivalently, every involution belongs to exactly one row of the following table.

(λ, β)	**Map $F(u, v)$**	**Allowed parameters**	**Fixed set**
$(1, 1)$	(u, v)	none	all of $\widetilde{M}$
$(1, -1)$	$(u, \alpha u - v + \delta)$	$\alpha, \delta \in \mathbb{R}$	$v = (\alpha u + \delta)/2$
$(-1, 1)$	$(-u + c, \alpha u + v - \alpha c/2)$	$c, \alpha \in \mathbb{R}$	$u = c/2$
$(-1, -1)$	$(-u + c, -v + \delta)$	$c, \delta \in \mathbb{R}$	$(u, v) = (c/2, \delta/2)$

Proof. Applying (8) twice gives

$$F^2(u,v) = \big(\lambda^2 u + (\lambda+1)c, \alpha(\lambda+\beta)u + \beta^2 v + \alpha c + (\beta+1)\delta\big). \tag{15}$$

Equality with (u,v) for every point is exactly (14). Since $\lambda, \beta \in \{1,-1\}$, solving the remaining three equations yields the four table rows. Solving $F(u,v) = (u,v)$ gives the listed fixed sets. □

5.1 The two elementary exchange maps

The plain coordinate exchange $E(x,y) = (y,x)$ becomes

$$E(u,v) = (u,-v). \tag{16}$$

It is a form-preserving reflection from the second row of Theorem 2. It does not preserve M_+ because it exchanges a nonnegative first coordinate with a nonpositive second coordinate without changing signs.

The signed exchange from FRC 830.000 is

$$D(x,y) = (-y,-x), qquad D(u,v) = (-u,v). \tag{17}$$

Its parameters are $(-1,0,0,1,0)$. It belongs to the third row, reverses ω, and has fixed set

$$\mathrm{Fix}(D) = \{u=0\} = \{x+y=0\}. \tag{18}$$

The entire zero-reciprocity leaf is fixed pointwise; the two sides of that leaf are exchanged.

6. The operational half-plane M

The domain M encodes $y = \ln C \le 0$ without assuming a sign for the entropy coordinate x
.

Theorem 3. Affine rigidity of the coherence half-plane

Every kernel-preserving affine automorphism $F : M \to M$ has the form

$$F_{\lambda,\rho,p}(x,y) = \big(\lambda x + (\lambda-\rho)y + p, \rho y\big), \qquad \lambda \ne 0, \quad \rho > 0. \tag{19}$$

Conversely, every map (19) is such an automorphism. It satisfies

$$F^*\omega = \lambda\omega, qquad q \circ F = \lambda q + p. \tag{20}$$

Proof. Write a general affine map as

$$x' = ax + by + p, qquad y' = rx + sy + t. \tag{21}$$

An affine automorphism of a closed half-plane maps its boundary line onto its boundary line. Since $y = 0$ must imply $y' = 0$ for every x, one has $r = t = 0$. Mapping the interior $y < 0$ onto itself requires $s > 0$. Invertibility requires $a \neq 0$.

Kernel preservation now says

$$(1, 1) \begin{pmatrix} a & b \\ 0 & s \end{pmatrix} = \lambda(1, 1), \tag{22}$$

so $a = \lambda$ and $b + s = \lambda$. Setting $s = \rho$ gives (19), and direct calculation gives (20). The same equations show that every listed map is an automorphism. □

Corollary 2. All involutions of M

Squaring (19) yields

$$F^2(x, y) = \left(\lambda^2 x + (\lambda^2 - \rho^2)y + (\lambda + 1)p, \rho^2 y\right). \tag{23}$$

Because $\rho > 0$, the involution condition forces $\rho = 1$. It then forces $\lambda = 1, p = 0$, giving the identity, or $\lambda = -1$ with arbitrary p, giving

$$R_p(x, y) = (-x - 2y + p, y). \tag{24}$$

Each R_p satisfies

$$R_p^* \omega = -\omega, qquad q \circ R_p = -q + p, qquad \mathrm{Fix}(R_p) = \{x + y = p/2\}. \tag{25}$$

Thus the half-plane admits a one-parameter family of sign-reversing involutions. The special map D is absent because D does not preserve all of M when $x < 0$.

7. The nonnegative-entropy cone M_+

The smaller domain M_+ contains the normalized entropy and negative log coherence quadrants used in many FRC applications.

Theorem 4. Domain rigidity and uniqueness of D

Every kernel-preserving affine automorphism of M_+ is exactly one of

$$F = aI \quad \text{or} \quad F = aD, qquad a > 0. \tag{26}$$

Consequently:

1. I is the unique automorphism satisfying $F^*\omega = \omega$;
2. D is the unique automorphism satisfying $F^*\omega = -\omega$; and
3. D is the unique nonidentity involution in this affine,

kernel-preserving, domain-automorphism class.

Proof. Use cone coordinates

$$r = x \geq 0, qquad s = -y \geq 0, \tag{27}$$

so M_+ is the closed positive quadrant. Its origin is its unique extreme point. An affine bijection of a convex set maps extreme points to extreme points, hence every affine automorphism of M_+ fixes the origin and has no translation.

The two coordinate rays are the two extreme rays of the cone. A linear cone automorphism must permute them. Therefore in (r, s) coordinates it is either

$$\begin{pmatrix} r' \\ s' \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} r' \\ s' \end{pmatrix} = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix}, \qquad a, b > 0. \tag{28}$$

Returning to (x, y), the two cases are

$$(x', y') = (ax, by) \quad \text{or} \quad (x', y') = (-ay, -bx). \tag{29}$$

In the first case, kernel preservation requires $(a, b) = \lambda(1, 1)$, hence $a = b = \lambda > 0$ and $F = aI$. In the second case it requires $(-b, -a) = \lambda(1, 1)$, hence $a = b = -\lambda > 0$ and $F = aD$. This proves (26).

Finally,

$$(aI)^2 = a^2 I, qquad (aD)^2 = a^2 I, qquad (aI)^*\omega = a\omega, qquad (aD)^*\omega = -a\omega. \tag{30}$$

Since $a > 0$, exact preservation, exact reversal, or involutivity forces $a = 1$ in the relevant branch. □

7.1 What the uniqueness theorem wins

The theorem upgrades the result of FRC 830.000. The map D is not an arbitrary sign choice. Within the explicitly bounded class—affine, invertible, kernel-preserving, and globally compatible with both $x \geq 0$ and $y \leq 0$—it is the only nontrivial involution and the only exact sign reversal of the one-form.

The theorem does not say that D is the only nonlinear transformation, the only local transformation, or the only map on an underlying physical state space. Its strength and its boundary are the same: it is a complete rigidity result for one declared coordinate class.

8. Offsets, coherence calibration, and units

8.1 Entropy offsets

An entropy-coordinate offset gives

$$T_s(x, y) = (x + s, y), qquadT_s^*\omega = \omega, qquadq \circ T_s = q + s. \tag{31}$$

It is an automorphism of M for every s. It is not an automorphism of M_+ unless $s = 0$: for $s < 0$ it sends some boundary points outside $x \geq 0$, and for $s > 0$ its image omits $0 \leq x < s$. This shows why an entropy-zero convention is invisible to the differential form but visible to the cone.

8.2 Multiplicative and power calibration of coherence

Let

$$C' = aC^\eta, qquada > 0, qquady' = \eta y + \ln a. \tag{32}$$

This maps $(0, 1]$ into itself exactly when $\eta > 0$ and $0 < a \leq 1$. It maps $(0, 1]$ onto itself exactly when $\eta > 0$ and $a = 1$. For $0 < a < 1$, the image is the proper interval $(0, a]$.

If x is held fixed, then

$$F^*\omega = dx + \eta\, dy. \tag{33}$$

This is proportional to $dx + dy$ only when $\eta = 1$. In that special case, $C' = aC$ changes y only by a translation, preserves the one-form, and moves q by $\ln a$. For $a < 1$ it is an into-map of M, not an automorphism.

The point is not that calibration is forbidden. It is that a calibration map, a form symmetry, a leaf symmetry, and a domain automorphism are different claims.

8.3 Lawful unit conversion and inadmissible mixed-register substitution

The canonical k^* is the scale-invariant Boltzmann bridge. Its numerical register representation changes together with the numerical entropy representation. For a lawful conversion with $a_{\nu\mu} > 0$, let

$$S_\nu = a_{\nu\mu} S_\mu, \qquad k^*_\nu = a_{\nu\mu} k^*_\mu, \qquad C_\nu = C_\mu. \tag{34a}$$

Then

$$x_\nu = \frac{S_\nu}{k^*_\nu} = \frac{S_\mu}{k^*_\mu} = x_\mu, \qquad y_\nu = y_\mu, \qquad dx_\nu + dy_\nu = dx_\mu + dy_\mu. \tag{34b}$$

Thus a lawful entropy-unit conversion is the identity on normalized ledger coordinates. It neither changes the affine symmetry class nor supplies a new physical transformation.

By contrast, an **inadmissible mixed-register substitution** holds a numerical entropy value from one register while replacing only the bridge representation, $k^*_{\text{new}} = k^*/\eta$. It gives

$$x' = \frac{S}{k^*_{\text{new}}} = \eta x, \qquad y' = y, \qquad dx' + dy' = \eta\, dx + dy. \tag{34c}$$

For the declared form $dx + dy$, this fails even kernel preservation unless $\eta = 1$. It is not a unit conversion. Changing k^* alone is not an internal symmetry and may never be selected after seeing an outcome.

Corollary 3. Unit-representation independence

The affine classification, involution conditions, and domain-rigidity results of Theorems 1--4 are unchanged under every lawful entropy-unit conversion.

Proof. Equation (34b) shows that the source and target normalized ledger coordinates and the one-form $dx + dy$ are identical. Therefore the affine maps, domains, pullback conditions, and involution equations classified in Theorems 1--4 are identical in the two unit presentations. □

9. Counterexamples and the boundary of the theorem

9.1 An affine shear that destroys reciprocity directions

In adapted coordinates, let

$$H(u,v) = (u+v, v). \tag{35}$$

Then $H^*\omega = du + dv$, which is not proportional to du. A tangent vector to a reciprocity leaf need not remain tangent. The shear is invertible and affine, but it lies outside Theorem 1 because it mixes v into u.

9.2 Form preservation without invertibility

The projection

$$P(u,v) = (u, 0) \tag{36}$$

satisfies $P^*\omega = du = \omega$ but destroys all v information and has no inverse. Therefore preservation of the one-form alone is insufficient for a duality or equivalence.

9.3 A nonlinear kernel-preserving diffeomorphism

The map

$$N(u,v) = (\sinh u, v) \tag{37}$$

is a nonlinear diffeomorphism of $\mathbb{R}^2$ and satisfies

$$N^*\omega = \cosh(u)\, du. \tag{38}$$

It preserves $\ker \omega$ with a nonconstant multiplier but is not affine. This proves that Theorem 1 is a bounded affine classification, not a complete classification of all diffeomorphisms.

9.4 Zero-leaf preservation without kernel preservation

The nonlinear diffeomorphism

$$Z(u,v) = \left(u(1+v^2), v\right) \tag{39}$$

maps $u = 0$ exactly onto itself. Yet

$$Z^*\omega = (1 + v^2)du + 2uv\,dv, \tag{40}$$

which is not proportional to du when $uv \neq 0$. A map can preserve the zero set of the ledger total while failing to preserve the reciprocity distribution away from that leaf.

9.5 The surviving lift obstruction

All transformations classified in Theorems 1-4 act on derived coordinates. They remain ledger symmetries until a map

$$\widehat{F} : X_\mu \longrightarrow X_\nu \tag{41}$$

is independently defined on FRC state or model spaces and shown to induce the ledger action while respecting the observables, dynamics, units, and boundaries. No such lift is constructed in this paper.

10. Relation to prior work

10.1 Onsager and Casimir

Onsager reciprocity constrains cross-coefficients in linear irreversible thermodynamics under microscopic reversibility; the Onsager-Casimir extension tracks time-reversal parity and magnetic fields. Theorems 1-4 classify affine maps of a two-coordinate ledger. They do not prove symmetry of transport coefficients, identify thermodynamic forces and fluxes, or derive microscopic reversibility.

10.2 Born reciprocity

Born's program makes position and momentum roles reciprocal. The map D exchanges normalized entropy and negative log coherence at the coordinate level, but this paper supplies no symplectic form, canonical bracket, quantization, or position-momentum interpretation. The term *Born reciprocity* therefore remains a comparison, not an FRC result.

10.3 Majid's self-duality and semidualisation

Majid's duality program names algebraic structures, maps between observables and states, quantum groups, representations, and semidualisation operations. Majid and Schroers test those operations on explicit three-dimensional quantum-gravity models and state the algebraic level at which self-duality holds. By comparison, Theorem 4 gives a domain-rigid involution of ledger coordinates. It supplies a map, a square, a fixed set, and an anti-invariant one-form, but no Hopf-algebraic or representation-theoretic lift.

The methodological lesson is precise: the next FRC question is not whether a visually pleasing involution exists—it does—but whether it acts naturally on independently defined structures that generate S and C.

11. Claim-status table and kill result

ID	Statement	Status	Review state
T1	Every invertible affine kernel-preserving map has normal form (8).	Proved theorem	Internal algebra check complete; independent review pending.
C1	Form, reversal, zero-leaf, and individual-leaf subclasses follow from (λ, c).	Proved corollary	Follows directly from T1.
G1	The maps form a group with composition (11) and inverse (13).	Proved proposition	Direct substitution.
T2	Conditions (14) and the four-row table classify all affine involutions in the ambient group.	Proved theorem	Internal algebra check complete; independent review pending.
T3	Equation (19) classifies the kernel-preserving affine automorphisms of M.	Proved theorem	Boundary-line proof; independent review pending.
C2	The only involutions of M are I and the family R_p.	Proved corollary	Exact square (23).
T4	The kernel-preserving affine automorphisms of M_+ are exactly aI and aD.	Proved theorem	Extreme-ray proof; independent review pending.
C3	D is the unique nonidentity involution and exact form reversal in the T4 class.	Proved corollary	Exact calculation (30).
O1	A classified map lifts to independently defined FRC operational structures.	Open problem	Assigned to FRC 830.002 and 830.003.
P1	D is a physical, categorical, Born, or Majid-style self-duality.	Not claimed	Requires O1 plus non-circular realization and physical interpretation.

The paper's mathematical win is C3: D is unique in the stated affine, invertible, kernel-preserving, domain-preserving class on M_+. This is more than a visual analogy.

The paper resolves its kill condition on the obstruction branch. Every survivor remains a coordinate transformation, and no new invariant beyond the declared ledger geometry has been lifted to underlying structures. Therefore this paper does **not** call the result a physical duality. The affine classification survives; promotion beyond coordinate level is withheld.

This is a productive obstruction, not a reason to abandon the 830 series. It reduces the candidates passed to later papers: D is the unique exact sign-reversing involution to test on M_+, while R_p is the corresponding half-plane family when entropy sign is unrestricted.

12. What this paper does not establish and what comes next

This paper does not establish:

- a universal physical law for $dS + k^* d \ln C = 0$;
- a physical exchange of entropy and coherence observables;
- a categorical equivalence between FRC registers;
- a symplectic, quantum, Hopf-algebraic, or representation-theoretic duality;
- a global classification of nonlinear diffeomorphisms;
- compatibility of D with the von Mises/Kuramoto distribution family;
- compatibility of D with purity, interference, transport, or other

operational coherence routes; or

- independent proof review.

FRC 830.002 must decide whether D or the wider ambient group induces valid maps between declared operational tuples $\mathcal{R}_\mu$. FRC 830.003 must test D on the underlying von Mises distributions and their natural and expectation coordinates, not merely on their $(S, \ln C)$ image. Failure to lift is an acceptable and publishable obstruction result.

Appendix A. Exact reproducibility record

All new results are algebraic and require no fit or numerical data.

For Theorem 1, let $e = (1, 1)^T$. Kernel preservation is checked by

$$A^T e = \lambda e. \tag{42}$$

The adapted-coordinate Jacobian of (8) is

$$J_F = \begin{pmatrix} \lambda & 0 \\ \alpha & \beta \end{pmatrix}, \qquad \det J_F = \lambda\beta. \tag{43}$$

Matrix multiplication of two such Jacobians and affine translation vectors gives (11). Solving (8) for (u, v) gives (13). Squaring (8) gives (15), whose five coefficient equations are (14).

For Theorem 3, boundary preservation forces the second row of the affine map to be $(0, \rho)$ with zero translation, and interior preservation forces $\rho > 0$. Equation (22) then supplies the remaining first-row coefficient.

For Theorem 4, the two matrices in (28) exhaust permutations and positive rescalings of the two extreme rays. Multiplication by the covector of $dx + dy$ forces the two ray scales to agree, giving (26). Squaring and pulling back the form gives (30).

These steps reproduce the complete proof. A separate independent mathematical review is still required for evidence promotion.

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