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What Would Make FRC Reciprocity a Duality? v0.1

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Abstract

FRC uses dS + k d ln C = 0 as an operational bookkeeping relation and proposes a wider open-system interpretation as a conjecture. This paper asks a prior mathematical question: what additional structure would make that reciprocity a duality? It separates five claim classes—coordinate transform, bookkeeping identity, balance law, duality, and self-duality—and supplies exact counterexamples showing that none implies the next. On the normalized ledger plane, the map D(x,y)=(-y,-x) is a sign-reversing involution satisfying D^2=id and D*omega=-omega for omega=dx+dy. On the operational domain y<=0 it is a self-map only when x>=0, and no present construction lifts it from derived ledger coordinates to independently defined FRC states, observables,*

> *dynamics, or operational registers. Present FRC therefore reaches an exact coordinate-level sign-reversing involution on a restricted domain, but not a physical, categorical, or Majid-style self-duality. The result is a terminology lock, a promotion test, and a research gate for the 830 series.*

0. Result in one paragraph

FRC presently has more than a metaphor but less than a physical duality. In normalized ledger coordinates, its reciprocity one-form possesses an exact sign-reversing involution on a restricted domain. That is a legitimate formal result. It does not yet exchange independently defined physical structures, does not act on the operational routes that produce coherence, and has not been derived from dynamics or experiment. The correct current classification is therefore: an operational bookkeeping relation, a conjectural open-system extension, and an exact coordinate-level symmetry whose physical lift is open. This paper fixes the vocabulary and the tests that any stronger claim must pass.

1. Question, scope, and governing status

The canonical FRC relation is

$$dS + k^* d \ln C = 0, \qquad k^* > 0, \quad C \in (0, 1]. \tag{1}$$

FRC 566.001 assigns two different statuses to (1). At a declared operational register, boundary, and unit convention, it is used as a bookkeeping identity and control law. The claim that the same form governs real open systems is a physical conjecture. FRC 826.829 preserves that distinction as part of the mathematical foundation.

The word *reciprocity* is not enough to upgrade either use into a duality. Onsager reciprocity, Born reciprocity, Hopf-algebra duality, Fourier duality, and the FRC entropy-coherence ledger are different mathematical statements. They may motivate one another, but shared language is not shared structure.

This paper asks only:

> *What data and exact tests would permit FRC reciprocity to be called a duality or self-duality without changing the status of its existing claims?*

The answer is a terminology charter, three propositions, one exact restricted sign-reversing involution, and three counterexamples. It is not empirical evidence for (1), a classification of every reciprocity-preserving transformation, or a claim about quantum gravity. The full linear and affine classification belongs to FRC 830.001.

The source hierarchy is as follows:

- FRC 566.001 fixes the canonical law, operational use, and conjectural status.
- FRC 566.030 supplies the exact von Mises/Kuramoto statistical-family result.
- FRC 700.777 requires every operational register and boundary to be declared.
- FRC 787.787 separates observed, target, latent, and fundamental Lambda

objects.

- FRC 826.829 records the status-labeled mathematical open problems.

Nothing below silently promotes a conjecture in those papers.

2. Starting objects

Fix one declared information-unit representation μ and suppress its register subscript in the local coordinates. Define

$$x = \frac{S}{k^*}, \qquad y = \ln C. \tag{2}$$

Since $C \in (0, 1]$, the minimal ledger domain is

$$M = \mathbb{R} \times (-\infty, 0]. \tag{3}$$

The ambient plane is $\widetilde{M} = \mathbb{R}^2$. Define the reciprocity one-form

$$\omega = dx + dy. \tag{4}$$

Along a differentiable process γ, equation (1) is equivalent, in this fixed representation, to

$$\gamma^* \omega = 0. \tag{5}$$

The integral leaves of $\ker \omega$ in the ambient plane are the lines $x + y = c$.

For a physical or computational realization at register μ, the actual object is not merely a point (x, y). At minimum it is a tuple

$$\mathcal{R}_\mu = (X_\mu, S_\mu, C_\mu, k^*_\mu, B_\mu), \tag{6}$$

where X_μ is the state or model space and B_μ declares the boundary, environment channels, units, and accounting convention. Its ledger form is

$$\omega_\mu = d(S_\mu/k^*_\mu) + d\ln C_\mu. \tag{7}$$

The canonical k^* is the scale-invariant Boltzmann bridge. The register subscript does not create a different physical object at each scale: k^*_μ is its numerical representation in the units declared by $\mathcal{R}_\mu$.

A **lawful entropy-unit conversion** from μ to ν uses one predeclared factor $a_{\nu\mu} > 0$ and transforms the numerical entropy and bridge together:

$$S_\nu = a_{\nu\mu} S_\mu, \qquad k^*_\nu = a_{\nu\mu} k^*_\mu, \qquad C_\nu = C_\mu.$$

Consequently

$$\frac{S_\nu}{k^*_\nu} = \frac{S_\mu}{k^*_\mu}, \qquad \ln C_\nu = \ln C_\mu,$$

so lawful unit-equivalent presentations induce the identity on the normalized ledger and preserve $dx + dy$ exactly. This paper proves its coordinate results in one normalized ledger fiber. It does not introduce a running k^*, a beta function, or dynamics on a space of scales.

Different routes may legitimately supply C_μ: a Kuramoto phase-order parameter, a von Mises mean resultant length, a quantum purity, an off-diagonal interference measure, or a platform-specific transport composite. FRC does not need to rederive those successful objects. It must cite them, declare their domains, and avoid pretending that they are automatically the same observable. The task of comparing and composing those registers belongs to FRC 830.002.

3. Terminology lock

The following five classes are ordered by additional required structure, not by prestige.

Claim class	Minimum content	What it does not supply by itself

Coordinate transform	A declared change of variables on one object, with its domain and inverse where relevant.	A new state, dynamics, conservation law, or dual object.
Bookkeeping identity	An equality imposed or defined at a declared boundary and operational register.	Independent physical necessity or a symmetry map.
Balance or conservation law	A relation derived from specified dynamics, symmetries, or boundary accounting.	An equivalence between two structured objects.
Duality	Two declared structured objects, a map or correspondence between them, a composition rule, and a specified preserved or sign-reversed structure.	Self-duality, empirical realization, or universality outside the stated class.
Self-duality	A duality from an object to an equivalent realization of its declared dual, with a repeat-application rule and a nontrivial invariant or anti-invariant.	Physical truth unless the mathematical objects and map are independently realized.

3.1 A minimal duality datum

Because different fields use *duality* differently, this paper does not impose one categorical definition on all of mathematics. It imposes a minimum audit format on FRC. A proposed FRC duality must declare a datum

$$\mathfrak{D} = (\mathcal{A}, \mathcal{B}, D, D^{\vee}, \mathcal{I}), \tag{8}$$

where:

1. $\mathcal{A}$ and $\mathcal{B}$ are specified structured objects;
2. $D : \mathcal{A} \to \mathcal{B}$ and $D^{\vee} : \mathcal{B} \to \mathcal{A}$ are

defined maps, functors, transforms, or correspondences of the claimed type;

1. the compositions $D^{\vee}D$ and $DD^{\vee}$ have a declared relation to the

relevant identity maps or equivalences;

1. $\mathcal{I}$ is specified structure that is preserved, covariantly

transformed, or sign-reversed; and

1. at least one realization is not created circularly by defining one channel

from the other through the desired invariant.

A self-duality additionally requires $\mathcal{A}$ to be equivalent to its declared dual and a rule such as $D^2 \simeq \mathbf{id}$ on the relevant equivalence classes. The symbol $\simeq$ must itself be defined.

3.2 Immediate classifications

The observed Lambda expression

$$\Lambda_{\text{obs}} = \Lambda_0 \ln C_{\text{obs}} \tag{9}$$

is a coordinate transform of a declared coherence observation. It is useful: it turns multiplicative changes of C into additive changes of Λ. But (9) does not create independent Lambda dynamics or a dual partner of C.

Likewise, an operational use of (1) can close a ledger, define a residual, and support a control target without being a conservation law. If a specified model independently derives the same differential relation from its dynamics and boundary accounting, that model-specific occurrence may be classified as a balance law. The equation can therefore occupy different classes in different realizations; its status must travel with its derivation.

4. Exact separations and counterexamples

4.1 Proposition 1: a zero one-form along a curve does not imply a duality

Proposition 1 (balance-law insufficiency). Let $I \subseteq \mathbb{R}$ be an interval, let $f : I \to \mathbb{R}$ be differentiable, and let c be a constant. The curve

$$\gamma_f(t) = (f(t), c - f(t)) \tag{10}$$

satisfies $\gamma_f^* \omega = 0$. This fact supplies neither a dual object nor a duality map.

Proof. Pull back (4):

$$\gamma_f^* \omega = d(f(t)) + d(c - f(t)) = f'(t)dt - f'(t)dt = 0. \tag{11}$$

The construction works for every differentiable f. It says that the curve is tangent to $\ker \omega$, or equivalently lies on one leaf $x + y = c$. No second structured object, map between objects, or composition rule appears. Therefore satisfaction of a balance relation is insufficient for a duality. □

This is not a criticism of balance laws. It is a type distinction: a conserved quantity constrains motion, while a duality relates structured descriptions.

4.2 Counterexample 1: a logarithm is not a duality

The map

$$h : (0, 1] \to (-\infty, 0], \qquad h(C) = \ln C \tag{12}$$

is a smooth bijection with inverse $h^{-1}(y) = e^y$. Calling y a Lambda coordinate changes the representation of one channel. It does not establish a second independently measured channel, a dynamics, or an exchange symmetry. Thus a useful nonlinear coordinate can remain only a coordinate transform.

4.3 Proposition 2: an exact sign-reversing involution exists on the ledger plane

Define on the ambient plane

$$D : \widetilde{M} \to \widetilde{M}, \qquad D(x, y) = (-y, -x). \tag{13}$$

Proposition 2 (ledger sign-reversing involution). The map D satisfies

$$D^2 = \mathrm{id}, \qquad D^*\omega = -\omega. \tag{14}$$

Consequently D preserves the distribution $\ker\omega$ and maps the leaf $x + y = c$ to the leaf $x + y = -c$.

Proof. Direct composition gives

$$D(D(x, y)) = D(-y, -x) = (x, y). \tag{15}$$

Moreover,

$$D^*dx = d(-y) = -dy, qquad D^*dy = d(-x) = -dx, \tag{16}$$

so

$$D^*\omega = -dy - dx = -\omega. \tag{17}$$

If $v \in \ker \omega$, then $(D^*\omega)(v) = -\omega(v) = 0$, proving kernel preservation. Finally, $x' + y' = -(x + y)$ proves the leaf statement. □

This is the first positive mathematical result of the 830 series. It is an exact involutive automorphism of $\ker \omega$ that reverses the sign of its defining one-form. It is also strictly weaker than a physical FRC self-duality.

4.4 Proposition 3: the physical-domain restriction

Proposition 3 (domain obstruction). The map D is not a self-map of all of $M = \mathbb{R} \times (-\infty, 0]$. It is a self-map of

$$M_+ = [0, \infty) \times (-\infty, 0]. \tag{18}$$

Proof. If $(x, y) \in M$, then the second coordinate of $D(x, y)$ is $y' = -x$. Membership of the image in M requires $y' \leq 0$, hence $x \geq 0$. When $x \geq 0$ and $y \leq 0$, the first image coordinate is $x' = -y \geq 0$ and the second is $y' = -x \leq 0$, so $D(M_+) = M_+$. □

Many thermodynamic entropies are nonnegative under their usual conventions, so M_+ is not empty or artificial. But differential entropy can be negative, entropy offsets can depend on representation, and the FRC corpus has not declared $x \geq 0$ for every register. The restriction must therefore be stated, not hidden.

Even on M_+, the coordinate formulas

$$x' = -\ln C, qquad C' = e^{-x} \tag{19}$$

do not show that x' is the entropy of a dual physical system or that C' is an independently measurable coherence of that system. Equation (19) only defines transformed ledger coordinates.

4.5 The lift obstruction

Call the following failure the **ledger-to-physics lift obstruction**:

> *A map on $(S/k^*, \ln C)$ is not a physical FRC duality unless it lifts to a map on independently declared state spaces, observables, dynamics, and boundaries whose induced action on the ledger is the proposed coordinate map.*

Formally, a candidate D requires some $\widehat{D} : X_\mu \to X_\nu$ and declared compatibility relations, for example

$$\frac{S_\nu \circ \widehat{D}}{k_\nu^*} = -\ln C_\mu, \qquad \ln(C_\nu \circ \widehat{D}) = -\frac{S_\mu}{k_\mu^*}, \tag{20}$$

with boundary and unit maps fixed independently. Present FRC work has not constructed such a $\widehat{D}$.

4.6 Counterexample 2: visual exchange is not enough

On $\widetilde{M}$, the coordinate swap $E(x, y) = (y, x)$ obeys $E^2 = \mathrm{id}$ and $E^*\omega = \omega$. Yet it generally sends the FRC domain into a region whose second coordinate is x, which need not be nonpositive. More importantly, it identifies an entropy coordinate and a log coherence coordinate merely because they are plotted on two axes. A symmetric diagram is not a physical exchange map.

4.7 Counterexample 3: circular closure is exact but evidentially empty

If one defines

$$C(S) = \exp[-S/k^*], \tag{21}$$

then

$$d \ln C = -\frac{dS}{k^*}, \tag{22}$$

and (1) follows identically. This is an exact realization of the ledger equation, but it cannot test reciprocity because the coherence channel was constructed from entropy. The same circularity would make (19) look physical without supplying an independent state-space lift.

The obstruction is not the use of established mathematics. Science should reuse successful objects. The obstruction is using the target equation to define both sides and then counting the resulting identity as evidence.

5. Current classification of FRC reciprocity

FRC object or statement	Current class	Exact status	Missing promotion condition
$\Lambda_{\mathrm{obs}} = \Lambda_0 \ln C_{\mathrm{obs}}$	Coordinate transform	Exact by definition.	Independent Lambda dynamics or a map to

			another structured object.
$dS_\mu + k^*_\mu d\ln C_\mu = 0$ used to close a declared ledger	Bookkeeping identity/control law	Operationally deployed.	Independent derivation from the realization's dynamics and boundary.
The same form derived in a specified physical model	Model-specific balance-law candidate	Conditional on an actual derivation; not supplied by notation.	Reproducible derivation and independent measurements of both channels.
$dS/d\ln C = -\kappa r$ on the von Mises family	Exact statistical-family identity	Proven in FRC 566.030.	A map on the family and its states; the identity alone is not a duality.
$D(x,y) = (-y,-x)$ on M_+	Restricted coordinate-level self-symmetry	$D^2 = \mathrm{id}$ and $D^*\omega = -\omega$ by Propositions 2-3.	Non-circular lift to operational states, observables, dynamics, and boundaries.
Universal open-system reciprocity	Physical conjecture	Open.	Predeclared cross-system tests with independent boundary accounting.
Majid-style or Born-style FRC self-duality	Open mathematical and physical problem	Not established.	Declared exchanged structures, dual maps, composition, invariants, representations, and physical realization.

The von Mises result deserves special care. For

$$C = r = \frac{I_1(\kappa)}{I_0(\kappa)}, \qquad S = \ln(2\pi I_0(\kappa)) - \kappa r, \tag{23}$$

FRC 566.030 proves

$$\frac{dS}{d\ln C} = -\kappa r. \tag{24}$$

At the natural-information register μ_{nat}, where $k^*_{\mu_{\mathrm{nat}}} = 1$, the point $\kappa r = 1$ is a stationary point of $Q = S + \ln C$. It is not a global duality transformation, a zero-current theorem, or evidence for a bath that is absent from the family. FRC 830.003 will test whether any candidate from 830.001 lifts naturally to the von Mises distributions, their natural parameter κ, and their expectation parameter r.

6. The FRC duality promotion ladder

The 830 series uses five promotion levels. Passing one level does not imply the next.

Promotion level	Required result	Present FRC status
P0 — vocabulary resemblance	Two domains use words such as reciprocity, order, exchange, or duality.	Passed, but carries no mathematical weight.
P1 — coordinate symmetry	A defined transformation acts on ledger coordinates with a composition rule.	Passed by D on the ambient plane and on M_+.
P2 — invariant geometry	The transformation preserves or reverses a specified form, kernel, metric, action, algebra, or other structure.	Passed for $D^*\omega = -\omega$ and $\ker \omega$ on the restricted ledger geometry.
P3 — structural lift	The map acts on underlying state spaces, observables, distributions, algebras, or operational registers and induces the ledger action.	Open; blocked by (20).
P4 — independent physical realization	At least one non-circular realization is derived or measured, with units and boundaries fixed before the result.	Open.

Accordingly, it is accurate to say:

> *FRC has a P2 restricted ledger sign-reversing involution. It does not yet have a P3 structural duality or P4 physical self-duality.*

For journal-facing use, the word *duality* must always carry its level and object: for example, “sign-reversing involution of the normalized ledger one-form” is acceptable; “FRC proves entropy-coherence self-duality” is not.

7. Relation to neighboring reciprocity programs

7.1 Onsager reciprocity is a coefficient symmetry

Near equilibrium, linear irreversible thermodynamics writes fluxes J_i in terms of thermodynamic forces X_j as

$$J_i = \sum_j L_{ij} X_j. \tag{25}$$

Onsager's reciprocal relations constrain the cross-coefficients under the assumptions behind microscopic reversibility. With parity and magnetic-field effects included, the Onsager-Casimir form has the schematic structure

$$L_{ij}(B) = \epsilon_i \epsilon_j L_{ji}(-B). \tag{26}$$

This is not equation (1). FRC may use standard irreversible thermodynamics and may investigate whether entropy and coherence channels enter a legitimate force-flux system, but the scalar cancellation $dS + k^* d \ln C = 0$ does not by itself imply $L_{ij} = L_{ji}$. "Reciprocity" in the two settings names different claims.

7.2 Born reciprocity exchanges position and momentum structures

Born's reciprocity program treats position and momentum on a more symmetric footing and seeks physics invariant under an exchange of the corresponding phase-space roles. Whether a later theory realizes that program is a separate question; structurally, the proposal identifies what is exchanged.

FRC currently compares entropy and log coherence in a ledger. It has not shown that they are conjugate variables, has no symplectic form making them a position-momentum pair, and must not use the phrase *Born reciprocity* as if that work were already done.

7.3 What Majid's program has that FRC does not yet have

Majid's representation-theoretic self-duality and later quantum Born reciprocity are mathematically more advanced on the duality axis. The relevant work does not stop at a balanced equation. It uses declared algebraic objects: observables and states, Hopf algebras, Poisson-Lie or braided structures, quantum groups, representations, and semidualisation maps.

Majid and Schroers' three-dimensional quantum-gravity analysis is especially instructive. Semidualisation is an algebraic operation, defined with quantum group methods, that interchanges position and momentum sectors. They identify specific models related by the operation, analyze their representations, and state the algebraic level at which a q-deformed model is self-dual. Those are the kinds of objects, maps, and levels that make a duality claim auditable.

FRC should learn four concrete habits from this program:

1. name the two objects being exchanged, not only the quantities appearing in

one equation;

1. define the map and calculate its square or composition law;
2. state exactly what is invariant, anti-invariant, covariant, or exchanged;
3. lift the map to states, observables, distributions, or representations and

not only to derived coordinates.

This comparison does not imply that FRC should reproduce quantum-group theory or invent Kuramoto synchronization again. A framework is allowed—indeed required—to cite and use established tools. Its responsibility is to identify the new connective claim and test it without circular definitions. For FRC, the new candidate is not the existence of Kuramoto order or thermodynamic entropy. It is the claim that independently defined entropy and coherence routes admit one boundary-aware reciprocity structure across declared registers.

7.4 What FRC contributes at its present stage

FRC's positive content is not limited to admitting losses. At present it contributes:

- a normalized entropy-log-coherence ledger whose residual can be calculated;
- explicit operational registers rather than one undefined universal

“coherence” observable;

- independent coherence routes that can be tested rather than reinvented;
- boundary-relative status discipline separating definition, exact identity,

model result, and physical conjecture;

- exact statistical-family results such as (24); and
- a program of cross-register, distribution-level, and open-system promotion

tests that can fail cleanly.

These are framework and methodological results. They become a stronger scientific contribution only when the connective claims yield results not already implied by the imported components and survive comparison with standard baselines.

8. Claim-status table

ID	Statement	Status	Basis or required test

C1	Equations (2)-(7) define the normalized ledger coordinates, domain, register, and one-form; lawful entropy-unit conversion leaves the normalized ledger unchanged.	Definition and exact unit-naturality identity	Fixed by this charter and the scale-invariant bridge discipline of FRC 566.001 and FRC 826.829.
C2	A duality claim in the 830 series must declare the datum in (8) and an independent realization.	Definition / governance rule	Terminology lock for later papers.
C3	Every curve (10) satisfies $\gamma_f^*\omega = 0$.	Proposition	Exact calculation (11).
C4	$D^2 = \mathrm{id}$ and $D^*\omega = -\omega$ on $\widetilde{M}$.	Proposition	Exact calculations (15)-(17).
C5	D is a self-map of M_+ but not of all M.	Proposition	Domain calculation in Proposition 3.
C6	A physical lift would require independently defined compatibility relations of the form (20).	Necessary-condition proposal	Must be specialized for each operational register.
C7	Present FRC has a P2 restricted ledger sign-reversing involution.	Formal classification	Follows from C4-C5 and the ladder definitions.
C8	Present FRC has a P3 or P4 duality.	Open problem; not claimed	Requires a non-circular structural lift and physical realization.
C9	The canonical relation universally governs real open systems.	Physical conjecture inherited from FRC 566.001	Requires independent cross-system tests and boundary accounting.
C10	All legitimate FRC coherence routes compose into one duality structure.	Open problem	Target of FRC 830.002; no composition law yet established.

9. Named obstructions and kill condition

Five obstructions must remain visible:

1. **Domain obstruction:** a candidate map may fail $C \in (0, 1]$ or another

operational domain, as D fails on $x < 0$.

1. **Ledger-to-physics lift obstruction:** a coordinate map may have no action

on states, observables, dynamics, or boundaries.

1. **Independence obstruction:** defining one channel from the desired relation

makes the test circular.

1. **Register obstruction:** Kuramoto order, purity, interference visibility,

and transport composites need not admit natural maps between them.

1. **Boundary obstruction:** a system-only equality may change when entropy

exchange and environmental coherence channels are included.

The series kill condition is:

> *If coordinate transforms, bookkeeping identities, balance laws, dualities, and self-dualities cannot be separated without contradicting the canonical FRC status hierarchy, the 830 namespace is incoherent and should remain an appendix to FRC 826.829.*

This paper does **not** trigger the kill condition. The five classes can be separated consistently, and the exact P2 result can be stated without promoting the P3-P4 claims. This conclusion licenses the research series; it does not validate the later duality conjectures.

10. Acceptance tests for the next papers

FRC 830.001 may advance only if it:

1. classifies a bounded, declared class of linear or affine maps;
2. distinguishes preservation of ω from sign reversal of ω and

preservation only of $\ker \omega$;

1. includes offsets, unit changes, calibration of C, and physical-domain

restrictions;

1. proves its classification and provides counterexamples outside it; and
2. calls the surviving maps coordinate symmetries unless a structural lift is

separately established.

FRC 830.002 must then test maps between operational tuples $\mathcal{R}_\mu$, including identities, composition, and independently meaningful coherence routes. FRC 830.003 must test surviving candidates on the underlying von Mises distributions rather than only their $(S, \ln C)$ image. FRC 830.004 must derive any connection, curvature, or holonomy claim from an explicit open-system path measure and boundary model.

A later physical-duality paper is admissible only after P3 and P4 are met in at least one realization. A negative obstruction theorem is an acceptable result at every stage.

11. What this paper does not establish

This paper does not establish:

- that (1) is a universal thermodynamic or open-system law;
- that entropy and coherence are conjugate variables;
- that the plane involution D is a map between physical systems;
- that FRC operational registers form a category, groupoid, or bundle;
- that k^* is a running coupling, independently evolving state, or

semidualised dynamical scale;

- that Λ_{obs} is a fundamental field or has independent dynamics;
- that the von Mises stationary point is self-dual;
- that FRC realizes Born reciprocity, Hopf-algebra duality, quantum-group

semidualisation, or quantum-gravity self-duality; or

- that citing established community methods weakens FRC. The scientific issue

is whether FRC adds a precise, independently testable connective claim.

Appendix A. Exact reproducibility record

No fit, simulation, or numerical data enter Propositions 1-3. They can be checked by the following matrix representation. Write

$$z = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \qquad D(z) = Az. \tag{27}$$

Then

$$A^2 = I, qquad A^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{28}$$

The first identity verifies $D^2 = \mathrm{id}$. The second verifies $D^* \omega = -\omega$, because $(1, 1)^T$ is the coefficient covector of ω. The domain test is the single inequality $-x \leq 0$, equivalent to $x \geq 0$.

For Proposition 1, multiply the coefficient covector by the tangent $\dot{\gamma}_f = (f', -f')^T$:

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} f' \\ -f' \end{pmatrix} = 0. \tag{29}$$

These checks reproduce every new formal result in the paper.

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